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Experiment on the physics of the PN junction

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Simple apparatus, suitable for an undergraduate laboratory, allows precise measurements of the forward characteristics of Si and Ge "transdiodes" at different temperatures in the range 150–300 K. The experimental results are used to obtain a fairly accurate value of the universal constant e/k (elementary charge to Boltzmann constant ratio) and of the energy gap of Si and Ge.

I. INTRODUCTION

A simple experiment on the physics of the PN junction may be carried out in undergraduate laboratory courses, providing a determination of both the universal constant e/k (i.e., elementary charge to Boltzmann constant ratio) and of the energy gap E_g of the semiconductor material the junction is made of. In the experiment we assume the junction to be well described by the ideal diode equation

$$I=I_0[\exp(eV/kT)-1], \quad (1)$$

where I is the current and V is the voltage applied to the junction, e is the elementary charge, k is the Boltzmann constant, T is the absolute temperature, and I_0 is the inverse current (i.e., the current extrapolated for large negative V values), that is strongly dependent on the temperature and on the energy gap E_g of the semiconductor material.

While real diodes only approximately obey Eq. (1), the ideal behavior is well followed[1] by transistors whose collector and base are kept at the same voltage (this configuration is commonly named *diode connected transistor* or *transdiode).* Therefore we will use transdiodes as the best approximation to ideal diodes.

The experiment consists in measuring the forward characteristic of Si and Ge transdiodes, at various constant temperatures in the range $150\text{ K} < T < 300\text{ K}$. At any given temperature the semilogarithmic plot of the collector current I_c vs the base-emitter voltage V, for $V \gg kT$, is a straight line from which we may extract two quantities of interest: its slope equals e/kT, so that, knowing the working temperature, we may obtain a value for the universal constant e/k, and the intercept gives the value of the inverse current I_0. The value of the energy gap for the transdiode semiconductor material may be derived from the temperature dependence of I_0.

In Sec. II we briefly recall the theoretical model that justifies Eq. (1), discuss the dependence $I_0(T,E_g)$ of the inverse current on the temperature and on E_g, as well as the temperature dependence of the energy gap $E_g(T)$, and we explain the procedure used to derive the energy gap.

In Sec. III we describe the experimental apparatus, and in Sec., IV we discuss the results obtained using two Si transistors and one Ge transistor.

II. THEORY

The current–voltage relationship of the ideal PN junction, originally derived by Shockley,[2] and described by Eq. (1), follows from the assumption that the total current is the sum of two contributions: a forward current $I_F = I_0 \exp(eV/kT)$ due to the majority carriers that overcome the junction potential barrier, and an inverse current I_0 due to the minority carriers. If V is the voltage of the anode (P) with respect to the cathode (N), the barrier height *decreases* with positive V values while it *increases* with negative values: this explains the rectifying behavior of the junction.

The current of majority carriers (electrons from N to P region, and holes from P to N region) depends exponentially on the voltage V applied to the junction, owing to the Boltzmann factor that gives the probability for a carrier to have an energy higher than the *effective* potential barrier across the depletion layer.

The inverse current is due to the thermally generated minority carriers that diffuse into the depletion layer, where they are accelerated by the local electric field. As long as V is not too large, I_0 depends only on the minority carriers' equilibrium concentration (n_{p0} for the electrons in the P region and p_{n0} for the holes in the N region), and on their diffusion rate. A simple model of this diffusion process,[3] gives for I_0 the following expression:

$$I_0=Ae\left(\frac{n_{p0}D_n}{L_n}+\frac{p_{n0}D_p}{L_p}\right), \quad (2)$$

where A is the junction area, D_n and L_n are the diffusion coefficient and diffusion length for the electrons, and D_p and L_p are the same quantities for holes.

Inserting into Eq. (2) the fundamental equation $p_0n_{p0}=p_{n0}n_0=(n_i)^2$ for the equilibrium concentrations, where n_i is the intrinsic carrier concentration, with the conditions $n_0\approx N_d$ in region N and $p_0\approx N_a$ in region P, gives

$$I_0=Ae\left(\frac{D_n}{L_nN_d}+\frac{D_p}{L_pN_d}\right)(n_i)^2, \quad (3)$$

where, N_a, N_d are the acceptor and donor concentrations in the P and N regions, respectively. It is well known[4] that the temperature dependence of the intrinsic carrier concentration n_i is given by

$$(n_i)^2=BT^3\exp[-E_g(T)/kT], \quad (4)$$

where B is a constant and $E_g(T)$ is the temperature dependent energy gap of the junction's semiconductor material. The weak temperature dependence of the diffusion terms in Eq. (3) may be approximated[5] as

$$Ae\left(\frac{D_n}{L_nN_a}+\frac{D_p}{L_pN_d}\right)\approx CT^{\gamma/2}, \quad (5)$$

where the constant γ depends on the semiconductor material.

Inserting Eqs. (4) and (5) into Eq. (3) yields the temperature dependence of I_0

$$I_0(T)=DT^{(3+\gamma/2)}\exp[-E_g(T)/kT], \quad (6)$$

where $D=BC$ is a constant.

In order to obtain an E_g value from measurements of $I_0(T)$, some assumption on the *temperature dependence* of E_g must also be made. It has been shown that $E_g(T)$ is closely approximated by a linear behavior at high temperatures (above 200 K)

$$E_g(T)=E_g^0-\alpha T. \quad (7)$$

The values of E_g^0 for Ge and Si, computed by Smith[6] from several sets of experimental measurements of $E_g(T)$ in a broad temperature range, are 1.205 eV for Si and 0.782 eV for Ge. At low temperature the measured E_g departs from Eq. (7), approaching a constant value. Therefore one must not confuse E_g^0 [i.e., the *linear extrapolation* of $E_g(T)$ to zero Kelvin] with the value E_0 of E_g *measured* at $T=0$ K. The accepted values for E_0 are in fact 1.170 eV for Si[7] and 0.746 eV for Ge.[8]

A function that reproduces the experimental values of the energy gap down to low temperature was suggested by Bludau *et al.*[9]

$$E_g(T)=E_0+C_1T+C_2T^2. \quad (7')$$

If we use the simple function [Eq. (7)] and take the logarithm of Eq. (6) we get

$$\ln I_0=[\ln D+\alpha/k]-E_g^0/(kT)+(3+\gamma/2)\ln T. \quad (8)$$

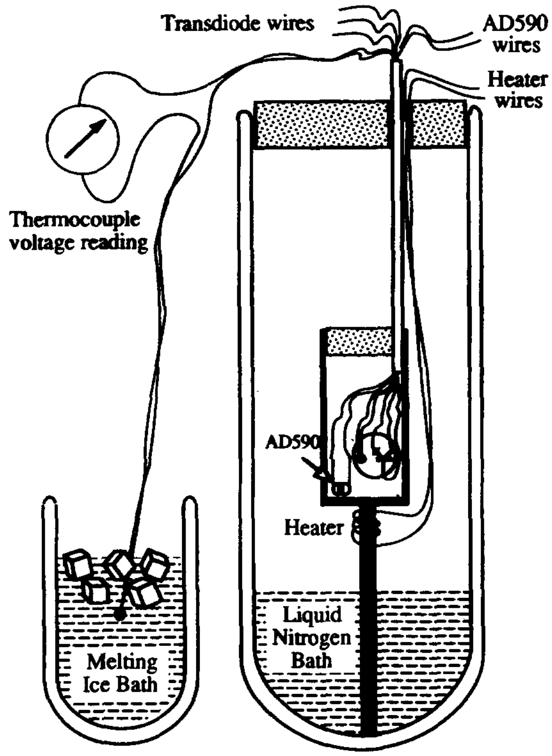

Fig. 1. The measuring cell.

This relation indicates that a semilog plot of I_0 vs $1/T$ is essentially a straight line, whose slope gives $-E_g^0/k$, because the term $(3+\gamma/2)\ln T$ introduces only a negligible curvature to the linear plot.

With respect to relation (7), relation (7') has the advantage of using the directly measurable parameter E_0 instead of the extrapolated one E_g^0 but it yields the more complicated dependence

$$\ln I_0=[\ln D-C_1/k]-E_0/(kT)-(C_2/k)T+(3+\gamma/2)\ln T. \quad (8')$$

III. EXPERIMENTAL APPARATUS

The transdiode is mounted in a small copper cell obtained from a thick walled tube, 2 cm diam, 5 cm long. The copper cell bottom is soft soldered to a brass rod which acts as a cold finger, when its low end is dipped into a liquid nitrogen bath. A 30 Ω constantan wire heater is wound around the upper end of the brass rod. An integrated circuit temperature transducer (AD590) is glued onto the cell bottom, to be used as a sensor for the thermoregulator that drives the heater. The temperature is measured by an iron–constantan (type J) thermocouple whose signal is read on a digital millivoltmeter. The thermocouple junction is thermally anchored to the sample by means of few turns of PTFE tape wrapped around the transistor case. All the electrical connections are made by thin wires fed into the cell through a thin walled stainless steel tube soldered to the copper cell. The cell is suspended by the steel tube inside a dewar vessel (see Fig. 1).

To measure the forward characteristics of the transdiodes we used a current–voltage converter that is an improved version of that originally used by Evans.[10] Here, by

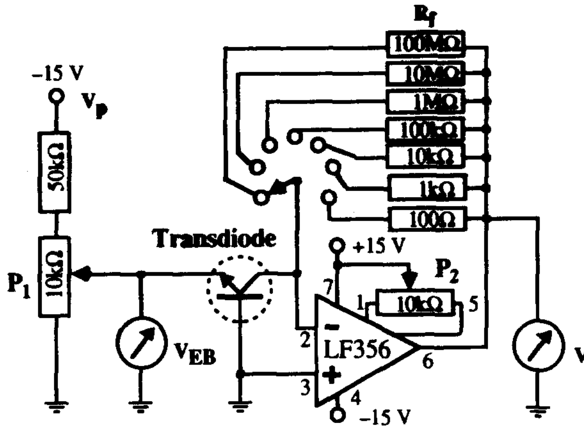

Fig. 2. The current–voltage converter used in the first experiment.

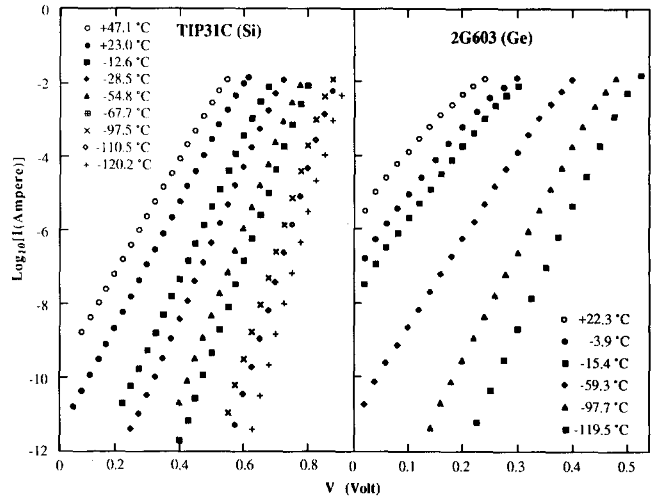

Fig. 4. Forward characteristics of Ge (2G603) and Si (TIP31C) transdiodes measured at constant temperatures.

substituting the general purpose operational amplifier (μA741) with a field-effect transistor input OA (LF356), we may reliably measure currents as small as 10^{-11} A. The inverting input of the OA (Fig. 2) is kept at virtual ground owing to the negative feedback and the high value of the OA open loop gain, so that the transistor is effectively operating in the transdiode configuration.

The value of the current–voltage conversion factor may be changed by switching on one of the seven feedback resistors R_f; these are 1% precision resistors accurately selected to match their nominal value. The output offset voltage is initially zeroed by adjusting the trimmer P_2 with $R_f = 100$ MΩ, and with the emitter short circuited to ground. Good shielding from pickup noise is achieved by enclosing the circuit into a metal box, and by connecting the transdiode through a coaxial cable.

The emitter-to-base voltage V_{BE} and the output voltage $V_0 = R_f I$ are measured, within ±0.1 mV, using two digital multimeters. With this circuitry we obtain an output drift stability of the order of 0.1 mV/h, and a current accuracy better than 10 pA in the most sensitive range. Figure 2 shows the setup for an NPN transistor; when a PNP transistor is used, the polarity of the bias voltage V_p must be reversed.

The sample is thermoregulated by means of the simple circuit shown in Fig. 3: the temperature sensor AD590[11] produces an output current of 1 μA/K, that is converted into a signal voltage V_T (−10 mV/K) by the current to voltage converter OA1. A stable adjustable reference voltage V_R is subtracted from V_T by a differential amplifier (OA2+OA3+OA4) with a differential gain in the range 1–20.

The temperature is set by trimming the potentiometer P_T (e.g., letting $V_R = -2$ V we get $T = 200$ K). The output voltage of the differential amplifier, which is proportional to the residual temperature offset, is amplified by an inverting amplifier (OA5: gain = 100) that drives the power transistor feeding current to the heater.

The time required to stabilize the temperature at the chosen value is of the order of 20–30 min so that, within a 3 h lab session, one may easily take five measurements of the forward characteristics at different temperatures.

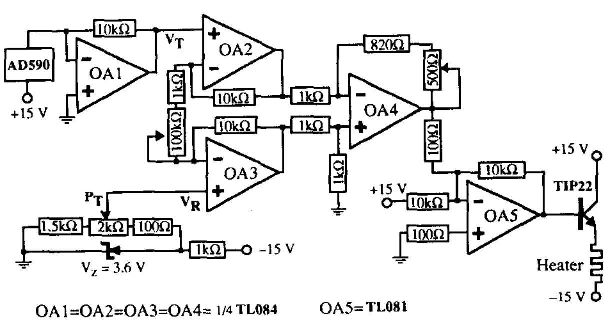

Fig. 3. The thermoregulator used in the first experiment.

IV. EXPERIMENTAL RESULTS

The forward characteristics, measured at several temperatures with two transdiodes (TIP31C:Si and 2G603:Ge), are reported in the semilogarithmic plots of Fig. 4, proving

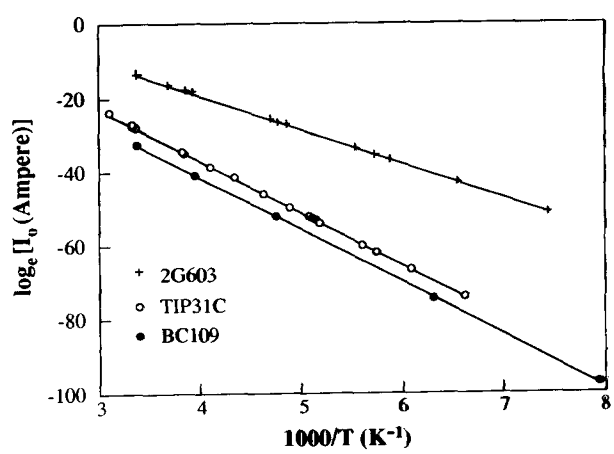

Fig. 5. The values of the inverse current I_0, extrapolated from the forward characteristics measured at various temperatures, plotted vs 1000/T. The lines represent the linear best fit.

Table I. Results for the energy gap *linearly extrapolated at* $T=0(E_g^0)$, and for the energy gap at $T=0$ (E_0).

E_g^0 (eV)	Transdiode	E_0 (eV)
1.223±0.011	TIP31C (Si)	1.17±0.02
1.217±0.018	BC109 (Si)	1.17±0.03
0.807±0.014	2G603 (Ge)	0.77±0.02

that the linear behavior predicted by Eq. (9) is obeyed, without appreciable deviation, in a very wide current range (i.e., from $I=10^{-11}$ up to 10^{-3} A).

The slight deviation from linearity at the lowest V values in the case of the Ge transdiode is due to the fact that the approximation $\exp(eV/kT)\gg 1$ fails in this range ($kT \approx 0.018$ eV at $T=210$ K). The departure from linearity in the high current range, on the other hand, is due to the effect of the finite resistivity of the bulk P and N semiconductors from which the junction is made. One might say that this deviation accounts for the ohmic voltage drop across the bulk that adds to the junction voltage.

From the values of the slope $S=\Delta[\ln(I)]/\Delta V$ of the forward characteristics in Fig. 4, obtained by a straight line least squares fit,[12] and the measured temperature T, we get the values for the ratio $e/k=ST$.

For example from TIP31C at $T=296.2\pm 0.2$ K we get $e/k=11\,618\pm 10$ CK J^{-1}, and from 2G603 at $T=210.6 \pm 0.5$ K we get $e/k=11655\pm 30$ CK J^{-1}, to be compared with the accepted value $e/k=11604.8\pm 0.6$ CK J^{-1}. The spread of the e/k values, obtained from all the other plots taken at different temperatures, is less than 2%, and it is almost completely due to the uncertainty in the temperature value.

The intercept with the $V=0$ axis of the logarithmic characteristic gives the value of I_0 at the working temperature: therefore from several runs performed at various constant temperatures we may get a measurement of the temperature dependence of $I_0(T)$, to be compared with the one predicted by Eq. (6).

The result of this procedure is shown in Fig. 5, where each point represents the I_0 value extrapolated from an isothermal run like those reported in Fig. 4. In order to reduce the main error source, we corrected the temperature values T_m, measured in each run, by using the accepted e/k value and the measured slope S of the $\ln(I)$ vs V plots: the corrected temperature is computed as $T_c = 11\,604.8/S$. The maximum change $\Delta T=T_c-T_m$ applied to our measurements by this correction is 2 K.

Figure 5 shows that the plot of $\ln I_0$ vs $1/T$ is essentially a straight line: this is the behavior predicted by relation (8) from which, by dropping the small term $(3+\gamma/2)\ln(T)$, we expect a slope $-E_g^0/k$.

By fitting the experimental points simply with a straight line we obtain for E_g^0 values (reported in Table I) that compare favorably with the accepted values of this parameter (see Table II). The errors are evaluated assuming $\Delta T=1$ K and $\Delta\ln(I_0)=0.1$.

Table II. Commonly accepted values of E_g^0 and E_0 (from Refs. 6–9).

E_g^0 (eV)	Material	E_0 (eV)
1.205	Si	1.170
0.782	Ge	0.746

The same approximation of neglecting the term in $\ln T$, was used by Collings:[13] this author however obtains E_g^0[Si] $=1.13$ eV, E_g^0[Ge]$=0.648$ eV probably due to less accurate measurements and/or to the use of normal PN junctions instead of a transdiodes.

We also obtained values for the E_0 parameter (energy gap at 0 K) by fitting our data with the function (8′) (but ignoring the term in $\ln T$). The results are reported in the third column of Table I. They are compatible with the accepted values, but they are obviously affected by a larger error due to the presence of one more free parameter in the fitting function. We consider unrealistic in our case to do a more complete analysis of the data including the logarithmic term and leaving the γ value as a free parameter, as made in a similar context by Kirkup and Placido.[14] In effect we observed that the fit becomes very sensitive to small changes of the experimental data, and it may yield unrealistically low values for E_g^0 when the minimum sum of squares corresponds to positive γ values.

V. CONCLUSIONS

The simple apparatus here presented allows the students to become familiar with most of the features of the PN junction physics, and with the fundamental concept of energy gap. It also allows a measurement of the universal constant e/k with an accuracy that is not usual in a teaching laboratory.

The data analysis here proposed helps clarifying the different meaning of two quantities (E_g^0 and E_0) that are frequently confused in the literature. Performing this experiment the students will also get a very useful technical hint: how to measure very small currents without sophisticated electronics.

[1]This point is explained in detail by G. B. Clayton, *Operational Amplifiers*, 2nd ed. (Butterworth, London, 1979), Chap. 5.3. He starts considering that the effective diode current is the sum of the diffusion current [Eq. (1)] plus various other terms (due to electron–hole generation in the depletion layer, surface leakage effects, etc.) that have the general form $I_j=I_{0j}[\exp(eV/m_jkT)-1]$, where the m_j parameters take values between 1 and 4. The transdiode behavior may then be explained, following the Ebers–Moll model [J. J. Ebers and J. L. Moll, "Large signal behavior of junction transistors," IRE Proc. **42**, 1761–1772 (1954)], by describing the transistor as two *interacting* PN junctions. The collector current is the sum of the terms: $I_{c0}[\exp(e|V_{CB}|/kT)-1]+\Sigma_j I_{c0j}[\exp(e|V_{CB}|/m_j\,kT)-1]$, due to the base–collector diode, plus the current $\alpha_F I_{E_0}[\exp(e|V_{EB}|/kT)-1]$ of the majority carriers of the emitter being injected into the base and diffusing to the collector (with only a small fraction $I-\alpha_F\approx 0.01$ being recombined with the base majority carriers). This last term is the only surviving when $V_{CB}=0$ (i.e., for base–collector short circuited), and therefore the transdiode exhibits an ideal diode behavior.

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[6]R. A. Smith, *Semiconductors* (Cambridge University, Cambridge, 1978), Chap. 13.3. See also the more recent review by O. Madelung, *Semiconductors: Group IV Elements and III–V Compounds*, in Data in Science and Technology, edited by R. Poerschke (Springer-Verlag, Berlin, 1991).

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[10]D. E. Evans, "Measurement of the Boltzmann's constant," Phys. Educ. **21**, 296–299 (1986). It must be noted that in the circuit reported by Evans the inverting and noninverting input channels of the AO are erroneously exchanged. See also F. W. Inman and C. E. Miller, "The measurement of e/k in the introductory physics laboratory," Am. J. Phys. **41**, 349–351 (1973).

[11]Analog devices gives for this transducer a useful linear range: -55, $+150$ °C. However when the linearity is not important, as in our case, it may be employed in a much wider temperature range: in our thermoregulator it was used down to 140 K.

[12]When necessary the slope and the intercept of the characteristic were obtained by fitting the experimental data to the complete Eq. (1) instead of using the straight line approximation.

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Infrared divergence, Thomson scattering, and Lamb shift

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The logarithmic part of the Lamb shift in hydrogen is related to the usual infrared divergence and can be obtained immediately from the corresponding radiative correction to the Coulomb scattering amplitude. By a simple generalization of Welton's arguments we get one of the recoil contributions. To complete the recoil calculation one has to include an elementary iteration of the Thomson amplitude. In this way we derive also the logarithmic part of the Lamb-shift operator in positronium and many-electron atoms.

I. INTRODUCTION

The explanation of the Lamb shift and precise calculation, together with precise measurement of the effect in various systems, certainly represent one of the triumphs of QED. However, although the subject has long ago entered numerous textbooks, its theoretical discussion as a rule lacks the intuitive component which is especially necessary in introductory courses. Even the direct relation of the logarithmic part of the Lamb shift to the infrared divergence remains rather obscure in those discussions.

We will demonstrate that a simple generalization of the intuitive picture of the Lamb shift given by Welton[1] allows one to obtain with logarithmic accuracy one of the recoil corrections to the effect. To complete the recoil calculation we include also the magnetic interaction, which is reduced to the iteration of the Thomson amplitude, and get in this way the logarithmic part of the Lamb Hamiltonian in positronium and many-electron atoms.

II. INFRARED DIVERGENCE AND LAMB SHIFT IN HYDROGEN

The origin of Lamb shift in hydrogen is closely related to the infrared divergence in the electron scattering from the Coulomb center. Indeed, using regularization via the introduction of a photon mass λ, the logarithmic λ dependence of the vertex part [Fig. 1(a)] is canceled by the analogous dependence of the Bremsstrahlung [Fig. 1(b)]. (We use the Coulomb gauge; the dashed line here and below refers to the Coulomb field, the wavy one to a transverse photon.)

If there is no acceleration, i.e., at vanishing momentum transfer q, the radiation vanishes also. Therefore, it is only natural that the infrared part of the vertex correction is proportional to q^2. Indeed, with account for this correction the potential of the electron interaction with a Coulomb center is, in the momentum representation (see, e.g., Ref. 2, paragraph 117),

$$V(\mathbf{q}) = -\frac{4\pi\alpha}{q^2}\left(1-\frac{\alpha q^2}{3\pi m^2}\log\frac{m}{\lambda}\right). \tag{1}$$

(We use the system of units with $\hbar=1$, $c=1$. The fine structure constant is $\alpha=e^2=1/137$ and m is the electron mass.) Of course, in the bound state problem there is no infrared radiation. But the electron here is not on the mass shell, the deviation from it coinciding in order of magnitude with the binding energy, $\sim m\alpha^2$. On the other hand, the role of the photon mass in the Bremsstrahlung process is, in fact, to fix the minimum possible deviation of the final state invariant mass from that of the free electron. So, in the bound state problem one can put $\lambda \to m\alpha^2$ in formula (1) to logarithmic accuracy.

Since the typical atomic momentum transfer is $q \sim m\alpha$, the relative magnitude of the correction to the potential, and that of the energy correction as well, is $\alpha^3 \log(1/\alpha)$.

More accurately, the discussed radiative correction to the potential to logarithmic accuracy is, in the momentum representation,

