Method for accurate resonant frequency measurements with a phase-modulated feedback loop

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A new method for accurate measurements of resonant frequency is described here, which uses a phase-modulated locked loop to avoid errors due to unknown phase shifts or to synchronous pickup noise.

INTRODUCTION

Accurate measurements of the resonant frequency can be performed using the phase-locking technique.[1-3] The standard phase-locking method uses a single feedback loop to catch and hold the resonant frequency. The accuracy of this method is limited mainly by the uncertainty on the zero phase reference (i.e., by the residual "phase error") and by the synchronous quadrature pickup.

A phase-modulated locking method is described here which uses a double feedback loop: the main loop is implemented by a second loop which allows a resonant-frequency measurement not affected by the residual phase and pickup errors.

We used this method to detect the adsorption of atomic layers onto a graphite fiber microbalance,[4] achieving an overall frequency resolution and stability $(\nu - \nu_0)/\nu_0 \leqslant 10^{-6}$.

I. THE STANDARD PHASE-LOCKED LOOP

In any resonant system the parameter that describes the oscillating response has amplitude and phase relationship with the excitation signal, both depending on the driving frequency.

Assuming the excitation signal as zero phase reference, the in-phase (V_{SF}) and the quadrature (V_{SQ}) components of the response signal V_S are given by the relation

$$V_S = V_{SF} + iV_{SQ} = V_{SO}\left[(\nu\Delta\nu)^2 + i\nu\Delta\nu(\nu_0^2 - \nu^2)\right]/\left[(\nu_0^2 - \nu^2)^2 + (\nu\Delta\nu)^2\right], \quad (1)$$

where ν is the driving frequency and $\Delta\nu$ is the resonance width. The behavior of V_{SF} and V_{SQ} close to the resonant frequency ν_0 is shown in Fig. 1.

If the measuring apparatus has a finite rejection of spurious signals $V_{PF} + iV_{PQ}$ which are synchronous with the excitation (such as electromagnetic pickup, mechanical feedthrough, etc.), the detected signal becomes

$$V_S = (V_{SF} + V_{PF}) + i(V_{SQ} + V_{PQ}) = V_F + iV_Q .$$

In the following analysis we will assume that close to ν_0 the frequency dependence of the pickup signal is negligible: $(\partial V_{PF}/\partial\nu) \approx (\partial V_{PQ}/\partial\nu) \approx 0$.

The slope of V_{SQ} at $\nu \approx \nu_0$ is negative and $V_{SQ}(\nu_0) \approx 0$. Therefore, if we measure V_S with a phase-sensitive detector driven at $\pi/2$ with respect to the excitation, the detector output can be used to lock at resonance the frequency of a voltage-controlled oscillator which produces the exciting signal.

An example of such a phase-locked loop is shown in Fig. 2 where a conducting wire W,[1,4] placed into a static magnetic field, is driven mechanically into vibration by a piezotransducer PT. The oscillation amplitude is detected by measuring the V_S voltage signal induced into the wire. The V_R quadrature signal for the reference channel of the phase-sensitive detector PSD1 is obtained from the driving signal V_D through a phase shifter PS.

The dc output signal V_1 of PSD1 yields the control for the voltage-controlled oscillator VCO, and the closed-loop frequency is measured by a frequency meter FM.

In this setup the closed-loop frequency may be shifted with respect to the resonant value, not only by the presence of a synchronous pickup, but also by the phase angle ϕ_0 between V_S and V_D introduced by the transducer PT and by the amplifier chain A_1.

The frequency error due to ϕ_0 can be reduced by a proper change of the V_R phase shift from $\pi/2$ to $\pi/2 + \phi_1$ with $\phi_1 \approx \phi_0$. Therefore, the "residual phase error" $\phi = \phi_1 - \phi_0$ can be manually minimized by searching for the maximum value of the rectified and filtered $\langle|V_S|\rangle$ signal with a dc voltmeter DCVM.

The accuracy of this technique can be predicted by the following simple calculations.

The PSD1 output can be written as a function of ν and ϕ

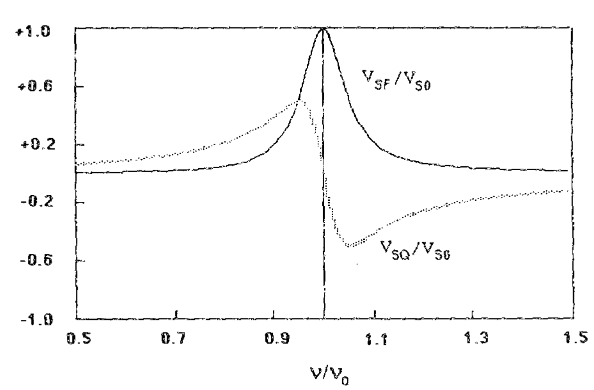

FIG. 1. The reduced amplitude of in phase (V_{FS}/V_{SO}) and quadrature (V_{SQ}/V_{SO}) signals at resonance.

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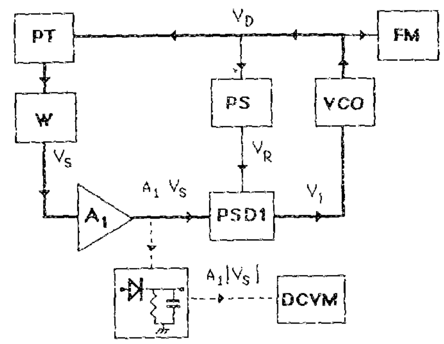

FIG. 2. Main feedback loop: PT piezotransducer; W vibrating wire; PS phase shifter; VCO voltage-controlled oscillator; FM frequency meter; PSD phase-sensitive detector.

$$V_1 = -A_1[(V_{SF}+V_{PF})\sin\phi + (V_{SQ}+V_{PQ})\cos\phi]$$
$$= A_1[\mathfrak{F}\sin\phi + \mathfrak{G}\cos\phi(\nu_0-\nu) + V_{PQ}\cos\phi]\,, \qquad (2)$$

where $\mathfrak{F}$ and $\mathfrak{G}$ are defined as

$$\mathfrak{F} = -V_{SO}[(\nu\Delta\nu)^2]/[(\nu_0^2-\nu^2)^2+(\nu\Delta\nu)^2] - V_{PF}\,,$$
$$\mathfrak{G} = V_{SO}[\nu\Delta\nu(\nu_0+\nu)]/[(\nu_0^2-\nu^2)^2+(\nu\Delta\nu)^2]\,. \qquad (3)$$

Let ν_w be the open-loop VCO frequency (i.e., the "free running frequency" for $V_1=0$), and A_0 the VCO sensitivity. The closed-loop frequency is then

$$\nu = \nu_w + A_0V_1. \qquad (4)$$

From Eqs. (2) and (4) we obtain

$$\nu = [\nu_w + A_0A_1\cos\phi(\mathfrak{G}\nu_0 + \tan\phi + V_{PQ})]/$$
$$(1+A_0A_1\mathfrak{G}\cos\phi)\,. \qquad (5)$$

The effect of the initial VCO setting (ν_w) disappears in the limit of infinite loop gain ($A_0A_1=\infty$)

$$\nu = \nu_0 + (\mathfrak{F}/\mathfrak{G})\tan\phi + V_{PQ}/\mathfrak{G}$$
$$= \nu_0 + \delta\nu(\phi) + \delta\nu(V_{PQ})\,. \qquad (6)$$

Equation (6) measures the shift of the closed-loop frequency with respect to the resonant frequency [note that it can be obtained directly from Eq. (2) letting $V_1=0$].

For $\nu\approx\nu_0$, Eqs. (3) yield $\mathfrak{F}\approx-(V_{SO}+V_{PF})$, and $\mathfrak{G}\approx 2V_{SO}/\Delta\nu$, so that Eq. (6) is approximated by

$$\nu\approx\nu_0 - \Delta\nu\tan\phi(V_{SO}+V_{PF})/(2V_{SO})$$
$$+\Delta\nu V_{PQ}/(2V_{SO})\,, \qquad (7)$$

or, using the quality factor $Q=\nu_0/\Delta\nu$,

$$(\nu-\nu_0)/\nu_0\approx -\tan\phi(1+V_{PF}/V_{SO})/2Q$$
$$+(V_{PQ}/V_{SO})/2Q\,. \qquad (8)$$

The smaller is Q, the larger the frequency shift.

To give an idea of the relevance of the two contributions to the total error we let, for example, $Q=10^2$, $V_{PF}\approx V_{PQ}\approx 10^{-2}V_{SO}$, and $\phi=5°$, obtaining $\delta\nu(\phi)/\nu_0\approx 4\times10^{-4}$ and $\delta\nu(V_{PQ})/\nu_0\approx 5\times10^{-5}$.

II. THE PHASE-LOCKED LOOP IMPROVED BY USING PHASE MODULATION

The residual phase error ϕ can be avoided if a second feedback loop is connected to the main loop by means of a voltage-controlled[5] phase shifter VCPS as shown in Fig. 3. Here the phase of the signal V_R fed to the reference channel of the main detector PSD1 is slightly modulated at a low frequency f ($f\ll\Delta\nu$ and $f\ll 1/\tau$, where τ is the PSD1 time constant) by the signal V_M produced by a second oscillator O.

Small changes of the driving frequency ν are, therefore, produced in the main loop by the modulated V_1 signal. This frequency modulation produces an amplitude modulation at frequency f of the rectified signal $|V_S|$, at the input of the second phase-sensitive detector PSD2. The second loop is closed by the output signal V_2 of PSD2 which is added to V_M at the VCPS input.

The zero phase error condition corresponds to $V_2=0$, i.e., $\partial|V_S|/\partial\nu=0$, if the effect of synchronous pickup is negligible.

A detailed analysis of the complete system, which accounts also for the pickup effect, is given in the following.

Let the VCPS be initially adjusted at open loop($V_2=V_M=0$) to give a reference signal V_R with a phase $\pi/2+\phi_1$ with respect to the exciting voltage. In this situation the circuits of Figs. 2 and 3 are equivalent and the main loop locks to a frequency given by Eq. (7).

When the modulating signal $V_M=V_{M0}\cos(2\pi f t)$ is switched on, the residual phase error $\phi(t)$ at closed loop is

$$\phi(t) = \phi_1 - \phi_0 + B_2V_2 + B_MV_M(t) = \langle\phi\rangle + B_MV_M(t)\,, \qquad (9)$$

where B_M and B_2 are the VCPS sensitivities for the V_M and V_2 inputs, respectively, and

$$\langle\phi\rangle = \phi_1 - \phi_0 + B_2V_2 \qquad (10)$$

is the mean value of the residual phase error (that can be assumed to be small). Therefore, $\tan\langle\phi\rangle\approx\langle\phi\rangle$ and Eqs. (7) and (9) yield the mean closed-loop frequency $\langle\nu\rangle$

$$\langle\nu\rangle = \nu_0 - \tfrac{1}{2}(1+V_{PF}/V_{SO})\langle\phi\rangle\Delta\nu + \tfrac{1}{2}V_{PQ}/V_{SO}\Delta\nu \qquad (11)$$

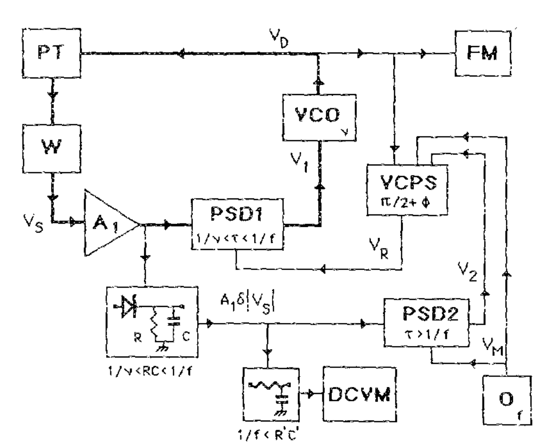

FIG. 3. Phase-modulated locked loop for phase error rejection: VCPS voltage-controlled phase shifter; O low-frequency oscillator.

and the frequency modulation

$$\delta\nu(t) = -\tfrac{1}{2}(1 + V_{PF}/V_{SO})\Delta\nu B_M V_{MO}\cos(2\pi f t)\,. \tag{12}$$

The amplitude of the rectified signal $|V_S|$ may be expressed as a function of the modulation $\delta\nu$

$$|V_S| = \langle|V_S|\rangle + \frac{\partial|V_S|}{\partial\nu}\delta\nu = \langle|V_S|\rangle + [\langle|V_S|\rangle]^{-1} \times\left(V_F\frac{\partial V_{SF}}{\partial\nu} + V_Q\frac{\partial V_{SQ}}{\partial\nu}\right)\delta\nu\,, \tag{13}$$

where the derivatives with respect to the frequency of the real and imaginary part are obtained from Eq. (1):

$$\frac{\partial V_{SF}}{\partial\nu} = V_{SO}\frac{2\nu(\Delta\nu)^2(\nu_0^2+\nu^2)(\nu_0^2-\nu^2)}{[(\nu_0^2-\nu^2)^2+(\nu\Delta\nu)^2]^2}\,, \tag{14}$$

$$\frac{\partial V_{SQ}}{\partial\nu} = V_{SO}\frac{\Delta\nu(\nu_0^2+\nu^2)[(\nu_0^2-\nu^2)^2-(\nu\Delta\nu)^2]}{[(\nu_0^2-\nu^2)^2+(\nu\Delta\nu)^2]^2}\,. \tag{15}$$

Close to the resonant frequency ($\langle\nu\rangle\approx\nu_0$), and for $V_{PF}\approx V_{PQ}\ll V_{SO}$ Eq. (13) reduces to

$$|V_S| = V_{SO} + 2/\Delta\nu[V_{SO}(\nu_0-\langle\nu\rangle)/\Delta\nu - V_{PQ}]\delta\nu\,. \tag{16}$$

The detector PSD2 produces an output voltage V_2 which is proportional to the amplitude modulation $\delta|V_S| = (\partial|V_S|/\partial\nu)\delta\nu$ of the rectified signal and, therefore, from Eqs. (12) and (16)

$$V_2 = A_2\delta|V_S| = -V_{SO}A_2B_MV_{MO}[2(\nu_0-\langle\nu\rangle)/\Delta\nu - V_{PQ}/V_{SO}]\,. \tag{17}$$

With an infinite loop gain the error voltage V_2 fed back to the VCPS is zeroed at closed loop. Letting $V_2 = 0$ into Eq. (17) we get

$$(\langle\nu\rangle-\nu_0)/\nu_0 = V_{PQ}/2QV_{SO}\,. \tag{18}$$

Equation (18) shows that the phase error appearing in Eq. (7) has been avoided by using the second loop.

The same result is obtained by introducing expression (17) of V_2 into Eq. (10)

$$\langle\phi\rangle = \phi_1 - \phi_0 - KV_{SO}[2(\nu_0-\langle\nu\rangle)/\Delta\nu - V_{PQ}/V_{SO}]\,, \tag{19}$$

where $K = A_2B_2B_MV_{MO}$ is the loop gain, and using expression (19) for $\langle\phi\rangle$ into Eq. (11), in the limit $K=\infty$.

III. FREQUENCY LOCK WITH HIGH REJECTION OF SYNCHRONOUS PICKUP

When the pickup error $\delta\nu(V_{PQ})$ is predominant with respect to the phase error $\delta\nu(\phi)$, the circuit shown in Fig. 4 should be used. Here the rectifier of Fig. 3 has been replaced by a third lock-in (PSD3), whose reference channel is fed by the driving signal V_D.

The output V_3 of PSD3 is $V_3(\nu) = [V_{SF}(\nu) + V_{PF}]\times\cos\phi + [V_{SQ}(\nu) + V_{PQ}]\sin\phi$, with $\phi=\phi_0$, where ϕ_0 is the (small) phase angle between V_D and V_S. The output V_2 of PSD2 is now

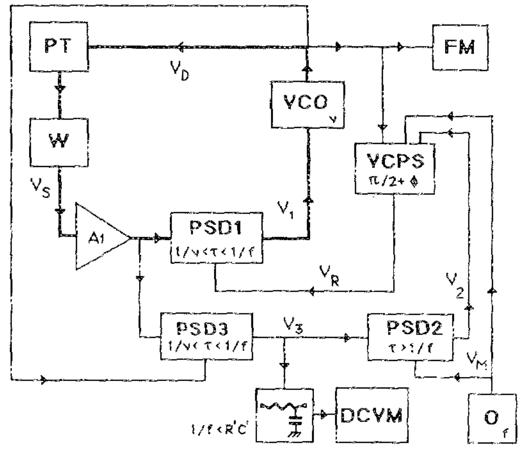

FIG. 4. Phase-modulated locked loop for synchronous pickup rejection.

$$V_2 = A_2\frac{\partial V_3}{\partial\nu}\delta\nu = A_2\left(\frac{\partial V_{SF}}{\partial\nu}\cos\phi + \frac{\partial V_{SQ}}{\partial\nu}\sin\phi\right)\delta\nu\,. \tag{20}$$

Using relations (12), (14), and (15) we get

$$V_2 = A_2B_MV_{MO}V_{SO}\cos\phi[2 + (1-\gamma)\tan\phi]\,, \tag{21}$$

where

$$\gamma = \frac{\nu_0^2-\nu^2}{\nu\Delta\nu}\,.$$

For $\nu\approx\nu_0$, V_2 becomes simply

$$V_2 = A_2B_MV_{MO}V_{SO}\cos\phi[\tan\phi + 2(\nu_0-\nu)/\Delta\nu]\,. \tag{22}$$

As usual, for large closed-loop gain we get $V_2\approx0$ and the frequency offset becomes

$$(\nu-\nu_0)/\nu_0 \approx \tan\phi/2Q. \tag{23}$$

IV. PERFORMANCE

The circuit shown in Fig. 3 has been used to perform adsorption measurements of argon on graphite close to the triple point.

The resonant frequency of the graphite fiber (1 cm long, 10 μm diameter) was in the range 10–20 kHz. Most of the synchronous pickup usually present in electrically driven vibrating wire was avoided by using mechanical excitation (piezoelectric ceramic).

The resonance width changes of three order of magnitude from vacuum to Ar saturated vapor pressure (the measured quality factor Q was $10^2 < Q < 10^5$). The phase error becomes important particularly at low Q values.

The phase error is due to changes of the excitation efficiency (e.g., coupling through the gas of various densities or temperature-dependent sound transmission coefficients) and to the electronics (e.g., finite bandwidth of the amplifiers and drifts with time and temperature). It was detected and zeroed by using the phase-modulated loop, driven at a frequency $f\approx4$ Hz ($Q<10^3$).

We obtained in this system an overall accuracy on the frequency lock better than $\Delta\nu/\nu = 10^{-6}$.

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