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Measurements of ^{4}He Viscosity Near the Superfluid Transition*

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The ^{4}He viscosity has been measured with a vibrating wire viscometer on the superfluid side of the λ transition. The results indicate that η should be singular at T_λ, in agreement with the suggestion advanced recently by G. Ahlers.

1. INTRODUCTION

Several sets of measurements of the viscosity of liquid ^{4}He have been recently analyzed by Ahlers[1] in order to establish whether or not η is singular at T_λ. He considers the function

$$1 - (\eta/\eta_\lambda) = A\varepsilon^x \tag{1}$$

where η_λ is the viscosity at the λ point and ε is the usual ratio $\varepsilon = (T_\lambda - T)/T_\lambda$. There are systematic differences in η among the results of the various experiments, but the analysis attained a good agreement for $1 - \eta/\eta_\lambda$. Over about two decades in ε the various measurements follow the relation $1 - \eta/\eta_\lambda = A|\varepsilon|^x$ with $A = 5.19$ and $x = 0.85$ for $T < T_\lambda$, and $A = -1.82$ and $x = 0.75$ for $T > T_\lambda$. Different values of η_λ were chosen for each set of data in the analysis. The conclusion was that the existing data strongly suggest the existence of a singularity in η at T_λ.

Webeler and Allen[2] later performed accurate measurements of viscosity in ^{3}He–^{4}He mixtures, using a torsional crystal method. They found no discontinuity in η or in $d\eta/dT$. Extrapolating the results to zero ^{3}He concentration, they concluded that η is regular at the transition.

An interesting result has recently been obtained for the ion mobility near T_λ.[3] This quantity, which depends on the viscosity η and on the surface tension σ,[4] shows a critical behavior as the λ point is approached.

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We report in this paper a set of measurements of the viscosity η in the λ region of pure ^{4}He. The measurements have been performed with a greatly improved version of the vibrating wire viscometer[5] first introduced by Tough *et al.*[6] Our cryogenic arrangement is not suitable for accurate measurements above T_λ and therefore we report only the data obtained below T_λ. Nevertheless the hypothesis suggested by Ahlers is strongly supported by our data.

2. EXPERIMENTAL METHOD AND APPARATUS

2.1. The Vibrating Wire Viscometer

We will give here only a short description of the method used for the measurement of the viscosity, together with relations that are peculiar to the special situation of He II. A detailed theory of the viscometer and a description of our technique can be found in Ref. 5. A tungsten wire of 5×10^{-3} cm diameter is stretched and clamped in a uniform and constant magnetic field B normal to the axis of the wire. The wire is driven to vibrate by a sinusoidal electric current of constant amplitude i_0 and variable frequency ν. The voltage induced in the wire circuit is amplified and detected by a phase sensitive detector. By changing the frequency ν of the driving current it is possible to get the resonance curve of the wire. The width $\Delta\nu$ of the resonance curve can be related to the density ρ and the viscosity η of the fluid in which the wire is vibrating. This relation can be developed with the aid of a linearized theory developed by Stokes.[7] In this theory the force per unit length exerted on the wire by the fluid is given by the formula

$$F = -\pi a^2 \rho[2\pi\nu k'(dy/dt) + k(d^2y/dt^2)] \tag{2}$$

where y is the displacement from the position at rest, a is the radius of the wire, and the numbers k and k' are known functions of the parameter m defined as

$$m = a/2\lambda = (a/2)(2\pi\nu\rho/\eta)^{1/2} \tag{3}$$

where λ is the viscous penetration depth. Introducing the force F into the equation of motion of the wire, one gets, with simple calculations,[5] the following useful equations:

$$\Delta\nu = \rho\nu_0 k'/(\rho_F + \rho k) \tag{4}$$

$$v_0 = A/\rho\nu_0 k' \tag{5}$$

where $\Delta\nu$ is the width of the resonance curve, ν_0 is the resonant frequency, ρ_F is the wire density, ρ is the fluid density, v_0 is the amplitude of the signal induced in the wire circuit at resonant frequency ν_0, and $A = 4 \times 10^{-9} l B^2 i_0/(\pi^4 a^2)$, with l the length of the wire in centimeters, B the value of the magnetic field in gauss, i_0 the driving current in amperes, and a the wire radius in centimeters. With these units, the induced signal v_0 at resonance is given by Eq. (5) in volts.

We can measure the viscosity η in two different ways. With the first method, after ν_0 and $\Delta\nu$ have been measured, we can choose a value m'' of the parameter m such that $k'(m'')$ and $k(m'')$ satisfy the relation (4). Once m'' is identified, and if the density ρ is known, the viscosity η is given by (3) as

$$\eta = \pi\rho a^2 \nu_0/(2m''^2) \tag{6}$$

This method allows an absolute measurement to be made, provided that some conditions on the amplitude of vibration are satisfied.[5]

In the second method we measure the resonant frequency ν_0 and the induced voltage v_0 at resonance. If ρ and A are known, we get $k'(m'')$ from (5) and the viscosity η from (6) as above. The constant A can be derived directly, or indirectly through measurements with a fluid of known density and viscosity. This method is particularly useful to detect small changes in viscosity.

In our technique the frequency of the driving oscillator is locked to that of the vibrating wire by means of a feedback loop connecting the output of the phase sensitive detector with the input of the voltage frequency control of the oscillator itself. If the reference of the phase sensitive detector is at $\pi/2$, the feedback locks the driving frequency to the resonant frequency ν_0. If the reference is set at $\pi - \pi/4$, the driving frequency is locked at a frequency ν_1, while for a reference at $\pi/4$ the locked frequency will be ν_2. The two frequencies ν_1 and ν_2 are such that the resonance width is given by $\Delta\nu = \nu_1 - \nu_2$. The feedback greatly improves the stability of the driving oscillator and the accuracy of the frequency setting. Moreover, it allows a measurement of the mean period to be made over long time intervals. These features greatly improve the accuracy of the measurements in both methods discussed above. The effects of the electronics dephasing and of stray pickup signals or resistive residuals are discussed in Ref. 5. Similarly, the effects of radiation damping, the limits on the vibration amplitude imposed by the linear approximation of the Stokes theory, the effects of the magnetic field alignment, and those arising from the fact that the wire is actually an anharmonic oscillator are all discussed in Ref. 5.

2.2. Application to He II

In the case of He II we must modify the Stokes formula (2) in order to take into account the two-fluid model. The term $\pi a^2 2\pi\nu\rho k'$ in the drag force (2) is the viscous coefficient, while $\pi a^2 \rho k$ is the hydrodynamic mass. Since the contribution to viscosity comes from the normal fluid only, we modify Eq. (2) as follows:

$$F = -\pi a^2[2\pi\nu\rho_n k'(dy/dt) + (\rho_n k + \rho_s)(d^2y/dt^2)] \tag{7}$$

We have used the fact that the contribution to the hydrodynamic mass given by the superfluid component is $\pi a^2\rho_s$. The other relations, deduced from (7) and from the equation of motion of the wire, have to be modified as follows:

$$\Delta\nu = \rho_n \nu_0 k'/(\rho_F + \rho_n k + \rho_s) \tag{8}$$

$$v_0 = A/\rho_n \nu_0 k' \tag{9}$$

$$\eta = \pi\rho_n a^2 \nu_0/2m''^2 \tag{10}$$

Our interest is to see how η changes with respect to its value η_λ at the λ point. For the ratio η/η_λ we have

$$\eta/\eta_\lambda = (\rho_n \nu_0/\rho_\lambda \nu_{0\lambda})(m''_\lambda/m'')^2 \tag{11}$$

With our apparatus $m \simeq 10$ in the λ region, and so the functions $k(m)$ and $k'(m)$ have the simple form[7]

$$k = 1 + \sqrt{2}/m \tag{12}$$

$$k' = \sqrt{2}/m + \tfrac{1}{2}m^2 \tag{13}$$

Combining (9), (11), and (13), we get

$$\eta/\eta_\lambda = (v_{0\lambda}/v_0)^2(\rho_\lambda \nu_{0\lambda}/\rho_n \nu_0)C(m'', m''_\lambda)$$

The coefficient C is a very slow function of m, so that we can take $C \simeq C_\lambda = 1$ for small changes in η. Finally we obtain the equation

$$\eta/\eta_\lambda = (\rho_\lambda \nu_{0\lambda}/\rho_n \nu_0)(v_{0\lambda}/v_0)^2 \tag{14}$$

The dissipation of the wire itself is taken into account by the usual relation $1/Q = 1/Q_T - 1/Q_0$,[5] which in the present case takes the form $1/v_0 = 1/v_m - 1/V_0$, where v_m is the measured voltage at resonance when the wire vibrates in helium, and V_0 is the corresponding voltage in vacuum. At helium temperatures the quality factor Q_0 in vacuum is of the order of 10^5, and $Q_\eta \simeq 10^3$; therefore the internal dissipation only results in a small correction to the measurements. It is to be noted that an accurate calculation of η by relation (8) or (14) from the measured ν_0 and $\Delta\nu$, or ν_0 and v_0, requires an accurate knowledge of the superfluid and normal densities ρ_s and ρ_n.

2.3. The Measuring Cell

The support for the vibrating wire is a small bar of lavite. Two brass screws are fixed on the bar, and the wire is stretched and clamped between two screwed nuts. The length of the wire is 2 cm, its radius is 2.5×10^{-3} cm, and its density is assumed to be 19.35 g/cm^3. The lavite bar is placed inside a copper box of rectangular cross section, which is soldered on a circular brass flange, as shown in Fig. 1. A germanium and a carbon resistor thermometer are located inside the cell. A permanent magnet is placed outside the cell in such a way that the produced magnetic field B is normal to the wire axis and parallel to the planes clamping the wire. With this arrangement we excite only one of the two vibration modes, as explained in Ref. 5. The closed cell and the magnet are mounted in such a way that the wire is horizontal, in order to avoid a possible gravitational effect. The expansion coefficient of tungsten and that of the supporting bar are very close in value. Therefore the resonant frequency changes only about 3% from room temperature to liquid helium temperature.

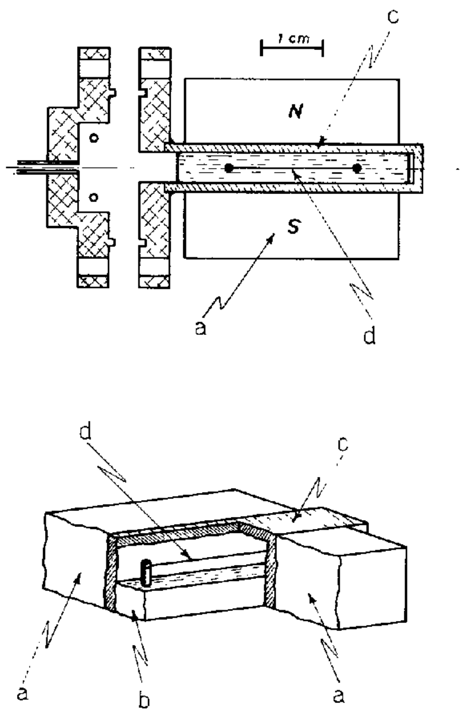

Fig. 1. Upper part: schematic view of the experimental cell. Lower part: sectional view of the viscometer. (a) Magnet poles, (b) insulating bar of lavite which supports the wire, (c) copper box, (d) tungsten wire.

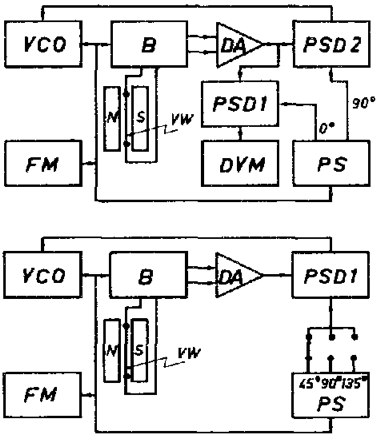

Fig. 2. Block diagrams of the electronics used in the relative method (upper part), and absolute method (lower part). VCO, voltage-controlled oscillator. B, bridge for suppression of stray signals or resistive residuals. DA, differential amplifier. PSD1, phase sensitive detector for measurement of the signal amplitude. PSD2, phase sensitive detector for frequency feedback. PS, phase shifter. DVM, digital voltmeter. FM, frequency meter.

2.4. Electronics

Figure 2 shows block diagrams of the electronic apparatus used for the absolute and relative measurements. A detailed description of the bridge B and of the phase selector PS can be found in Ref. 5. The phase sensitive detector PSD1 is a PAR lock-in amplifier model 116. In the first method it is used to lock the variable-frequency oscillator VCO to the wire frequency. In the second method it is used to amplify the induced signal, while the feedback on the frequency is performed by the home-made phase sensitive detector PSD2. Frequency and voltage are measured with digital instruments. A discussion of stability and accuracy can be found in Ref. 5.

2.5. Temperature Control and Measurement

The temperature was measured with the germanium thermometer (Leico Industries Model 3L) submerged in liquid helium inside the copper cell. The resistance of the thermometer was measured with a four-terminal bridge (Fig. 3). The reference resistor ($R_n \simeq 71\ \text{k}\Omega$) was outside the cryostat and thermoregulated at 30°C. Relative changes $\Delta R_n/R_n$ were smaller than 2×10^{-6}. The bridge is balanced using the variable ratio transformer T_3 (General Radio type 1493). The oscillator, amplifier, synchronous filter, and phase sensitive detector are home-made electronics.

The thermometer was calibrated by means of the Helium vapor pressure on the 1958 ^{4}He scale. The inaccuracy in the absolute temperature was estimated to be about 10^{-3} K. The value of the transformer ratio a that balances the bridge was a linear function of the temperature, to within an excellent approximation, from 2.0 to 2.2 K. The sensitivity $R^{-1}(\Delta R/\Delta T)$ was about $1.8\ \text{K}^{-1}$. With a dissipation of 3×10^{-8} W, changes ΔT of

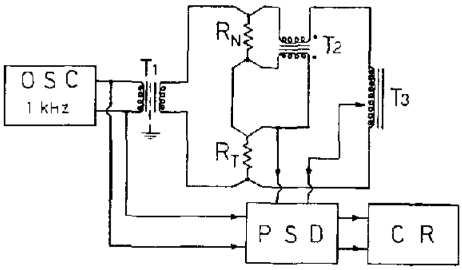

Fig. 3. Four-terminal bridge used in the measurement of germanium resistance. R_N, fixed resistor. R_T, thermometer. T_3, variable ratio transformer. T_2, transformer for reduction of changes in cable resistance.

10^{-5} K, corresponding to $\Delta R/R \simeq 2 \times 10^{-5}$, can easily be measured with our apparatus.

A first thermoregulation of the helium bath was achieved with a pressostat. The temperature of the sample cell was stabilized by means of a usual thermoregulator. As sensing element we used the carbon resistor inside the sample cell. The heater was wound directly around the copper part of the cell. This arrangement provided good temperature stability below T_λ. For temperatures above T_λ, however, the stability was not sufficient to improve the accuracy with respect to the existing data. Improved cryogenics need to be arranged for future work above the λ point.

2.6. Determination of the Lambda Point

The following procedure was used. With the bath thermoregulated we reached a temperature near T_λ. Then the bath temperature was released to drift very slowly across the λ point. As T_λ was approached, the drift was suppressed by the strong rise of the specific heat. Figure 4 shows a record of the temperature drift detected by the germanium thermometer. The temperature at which the germanium resistance was constant was assumed as T_λ. The maximum error due to the noise is $\pm 4\,\mu$K. A time interval of several minutes was largely sufficient to measure the quantities denoted by the subscript λ in the previous formulas.

3. EXPERIMENTAL RESULTS

3.1. Experimental Conditions

The measurements were made under the following conditions. The cell was filled with UPP helium gas (Società Italiana Ossigeno, Milan, Italy)

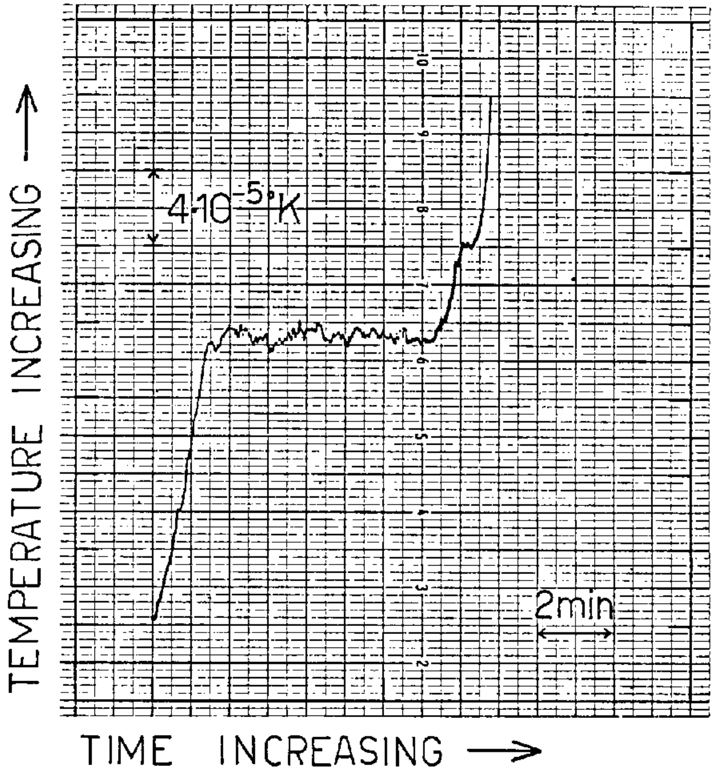

Fig. 4. A typical λ-point crossing, as recorded by the thermometer bridge. The λ temperature is defined within $\pm 4\ \mu$K.

passing through a charcoal trap cooled at liquid nitrogen temperature. The wire was positioned horizontally in a magnetic field of about 1000 G and was driven with a sine wave current of a zero peak amplitude $i_0 = 2.2 \times 10^{-5}$ A. The resonant frequency was $\nu_0 = 1792$ Hz, while the resonance signal amplitude changed from 18.5 μV at 2.03 K to 7.2 μV at T_λ. The mean power $\bar{w}_\eta$ dissipated in the fluid was therefore less than 5×10^{-10} W, and the Joule heating power $\bar{w}_J$ can be neglected with respect to $\bar{w}_\eta$ at our temperatures.

We will discuss now whether corrections must be introduced in the calculations of viscosity η. We use considerations and formulas developed in our previous paper.[5]

Power radiated by sound $\bar{w}_R$. The ratio of $\bar{w}_R$ to $\bar{w}_\eta$ is given by $\bar{w}_R/\bar{w}_\eta = 2\pi^3\nu_0 a^2/c^2 k'$. Taking for the velocity of sound c the value $c_\lambda = 21{,}790$ cm/sec at T_λ,[8] we get $\bar{w}_R/\bar{w}_\eta \simeq 10^{-8}$. Therefore no correction is needed for radiation damping.

Compressibility effects. The theory of the oscillating cylinder in incompressible fluid is valid also for a real fluid if $(\dot{y}_0/c)^2 \ll 1$ and if $(2\pi\nu a/c)^2 \ll 1$. The maximum value of the velocity $\dot{y}_0$ at the middle of the wire is given by $\dot{y}_0 = 10^8\pi v_0/2lB$. With $B = 10^3$ G, $v_0 = 18\ \mu$V, and $l = 2$ cm, we have $\dot{y}_0 = 1.4$ cm/sec, and then $(\dot{y}_0/c)^2 = 4 \times 10^{-9}$. We have also $(2\pi\nu a/c)^2 = 2 \times 10^{-6}$. The theory developed by Stokes for incompressible fluids is therefore a very good approximation for our case.

The linear approximation. Neglecting nonlinear terms in the Navier–Stokes equation imposes that $y_0/a \ll 1$. The maximum displacement y_0 can be calculated when i_0 and v_0 are known.[5] In the conditions of our experiment $y_0/a \simeq 5 \times 10^{-2}$ at $T = 2.03$ K and $y_0/a \simeq 3 \times 10^{-2}$ at T_λ. Therefore the linearized theory should be a good enough approximation.

3.2. Measurements with the First Method

Measured values of η obtained with the first method at $T < T_\lambda$ are shown in Fig. 5 (open circles), together with the data of Dash and Taylor[9] (filled circles) and Webeler and Hammer[10] (squares). As explained above, the data have been obtained by measuring the quality factor $Q_T = \nu_0/\Delta\nu$, where ν_0 is the resonant frequency and $\Delta\nu$ is the width of the resonant curve.

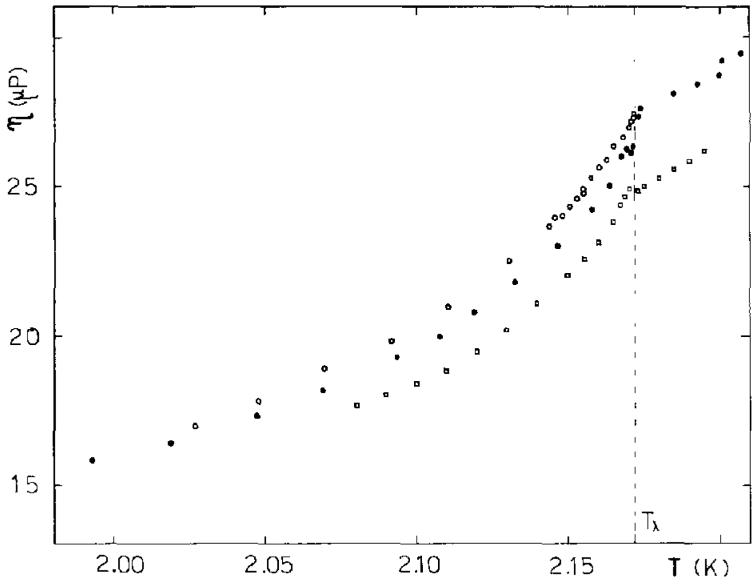

Fig. 5. The viscosity of ^{4}He above 2.0 K. Filled circles are data from Dash and Taylor,[9] squares are from Webeler and Hammer,[10] and open circles are from the present experiment (absolute method).

TABLE I

Quantities Involved in the Data Obtained with the First Method[a]

T, K	ρ, g/cm^3	ρ_n, g/cm^3	Q_η	η, μP
2.172000	0.146165	0.146165	882.7	27.38
2.171931	0.146164	0.145810	884.5	27.33
2.171719	0.146163	0.145261	886.8	27.29
2.171594	0.146162	0.145009	888.2	27.26
2.171374	0.146160	0.144623	889.9	27.22
2.170925	0.146157	0.14395	893.9	27.11
2.170704	0.146155	0.14366	895.2	27.08
2.170303	0.146153	0.14316	898.8	26.96
2.168066	0.146139	0.14092	911.5	26.62
2.165812	0.146127	0.13906	923.4	26.29
2.163631	0.146117	0.13747	935.3	25.92
2.160548	0.146103	0.13549	947.4	25.63
2.158384	0.146093	0.13416	958.5	25.29
2.155776	0.146082	0.13267	970.6	24.94
2.155715	0.146082	0.13262	974.4	24.76
2.153366	0.146073	0.13137	981.9	24.62
2.151090	0.146064	0.13017	991.9	24.34
2.148897	0.146056	0.12907	1003.5	23.99
2.146280	0.146046	0.12781	1007.5	24.03
2.144034	0.146038	0.12674	1019.1	23.68
2.131088	0.145997	0.12116	1068.7	22.53
2.110969	0.145954	0.11355	1143.8	20.99
2.091994	0.145898	0.10709	1212.8	19.80
2.070281	0.145842	0.10024	1282.8	18.89
2.048992	0.145786	0.09415	1362.3	17.83
2.027637	0.145741	0.08841	1440.3	16.97

[a]For symbols and errors see text.

From the values of Q_T we derived that of Q_η through the usual relation $1/Q_T = 1/Q_\eta + 1/Q_0$. The factor Q_0, measured with the wire vibrating in vacuum, changed from 80,500 at 4.2 K to 95,000 for $T \simeq T_\lambda$. The values of η have been calculated by means of the relations (8), (10), (12), and (13), using the experimental values of Kerr and Taylor[11] for the total density ρ and the experimental values of Tyson and Douglass[12] for the superfluid density ρ_s. We supposed that the wire was a cylinder with a radius a equal to the nominal value of 2.5×10^{-3} cm. The values of temperature, density, measured Q_η, and calculated viscosity are listed in Table I. Apart from a systematic error in the wire diameter, the error on the final data is about 0.1%.

3.3. Measurements with the Second Method

At temperatures very near T_λ, we used the second method, by which k' is obtained from measurements of the resonant frequency ν_0 and the

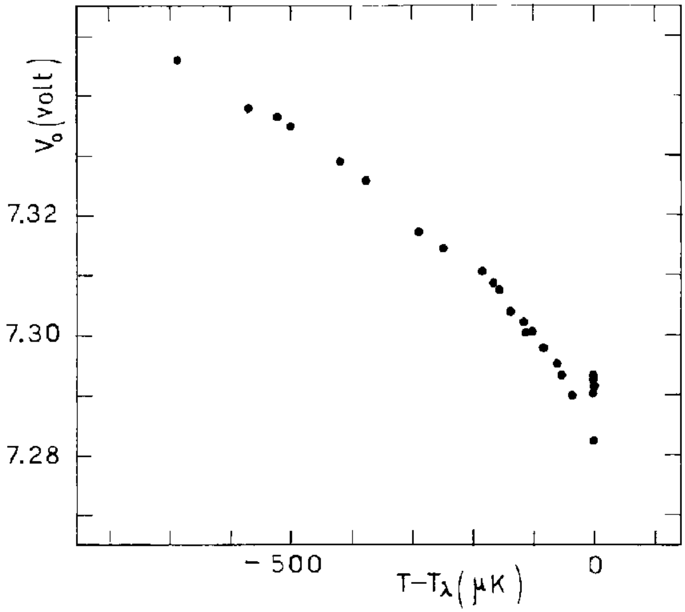

Fig. 6. The output voltage of phase sensitive detector PSD1, which amplifies the wire signal at resonance. It is measured with an accuracy of $\pm 3.5 \times 10^{-5}$. Small changes in amplifier gain, driving current, and stray signals scatter the points within $\pm 1.0 \times 10^{-4}$.

voltage amplitude v_0 at resonance. This voltage, amplified with highly stable electronics, was measured within a few parts in 10^5. Results of our measurements for $T_\lambda - T < 700\,\mu$K are shown in Fig. 6. The data were taken on increasing and decreasing the temperature, and the results were reproducible. Only at T_λ did we obtain several different values, while the germanium thermometer indicated a temperature well defined within $\pm 4\,\mu$K. The spreading of the data at T_λ is beyond the errors, but we cannot state that η is discontinuous at T_λ. Thermal stability above T_λ was not in fact good, and measurements were too scattered to be compared with those below T_λ. We hope to clarify this point in future work, using improved cryogenics. Values of η calculated from the data of Fig. 6 from the relation $\eta = A''(\rho_n v_0 v_0^2)^{-1}$ are plotted in Fig. 7. The normalization constant has been chosen as $A'' = 0.37625$. Values of the various quantities involved in Fig. 7 are listed in Table II.

4. DISCUSSION

Because the value of the viscosity η_λ at T_λ is not well defined, we must use an extrapolated value for η_λ in the plot suggested by Ahlers.[1] We found

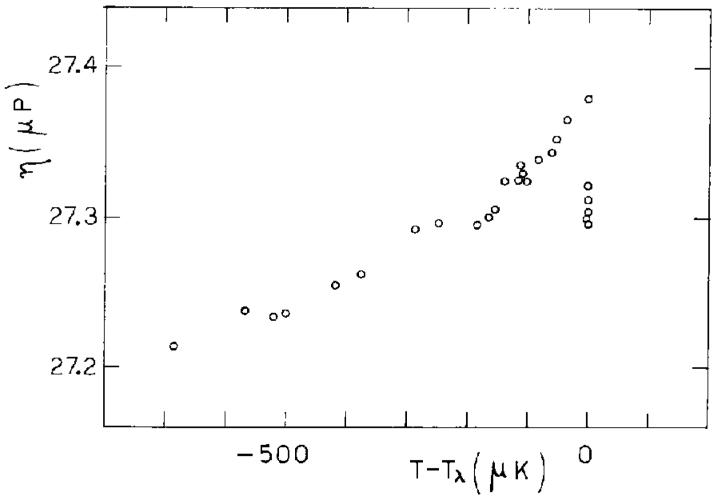

Fig. 7. The viscosity of He II calculated from the data of Fig. 6 as explained in the text. The temperature differences are in microdegrees.

TABLE II

Quantities Involved in the Measurements Performed with the Second Method[a]

$T_\lambda - T$, μK	ρ_n, g/cm^3	V_0, V	ν_0, Hz	η, μP
0	0.146165	7.2935	1772.80	27.296
112	0.145676	7.3006	1772.80	27.335
0	0.146165	7.2914	1772.80	27.312
0	0.146165	7.2925	1772.80	27.304
185	0.145481	7.3108	1772.80	27.295
164	0.145534	7.3088	1772.80	27.300
138	0.145603	7.3039	1772.80	27.324
108	0.145687	7.3011	1772.80	27.329
99	0.145714	7.3011	1722.79	27.324
82	0.145768	7.2979	1772.78	27.338
52	0.145872	7.2935	1772.76	27.352
34	0.145944	7.2899	1772.74	27.365
0	0.146165	7.2826	1772.74	27.379
0	0.146165	7.2935	1772.74	27.297
0	0.146165	7.2904	1772.70	27.321
59	0.145846	7.2954	1772.70	27.343
154	0.145560	7.3077	1772.70	27.305
115	0.145667	7.3023	1772.70	27.325
248	0.145333	7.3145	1772.70	27.296
287	0.145248	7.3172	1772.70	27.292
374	0.145071	7.3257	1772.70	27.262
417	0.144989	7.3289	1772.70	27.254
499	0.144839	7.3351	1772.70	27.236
520	0.144802	7.3364	1772.71	27.233
568	0.144720	7.3380	1772.71	27.237
684	0.144529	7.3460	1772.71	27.213

[a]For symbols and errors see the text.

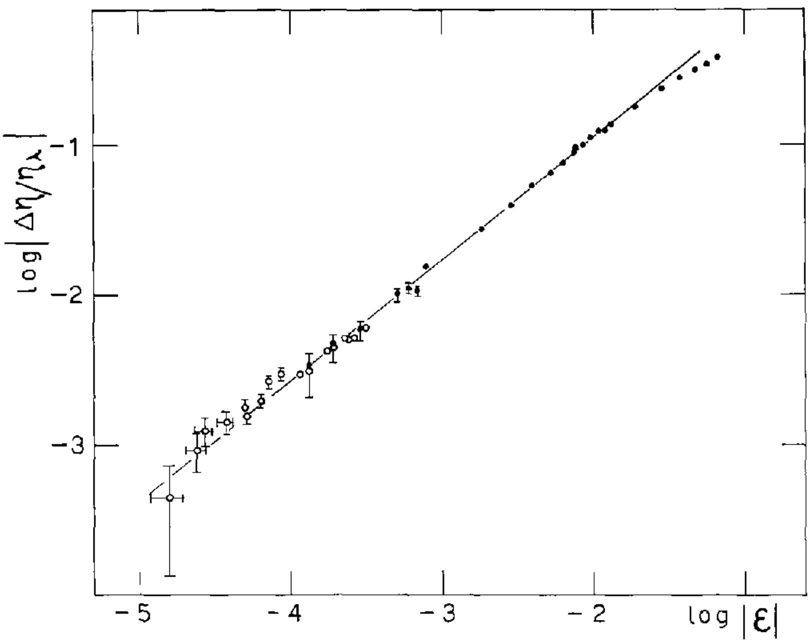

Fig. 8. The data of our experiment plotted as $\log[(\eta_\lambda - \eta)/\eta_\lambda]$ as a function of $\log[(T_\lambda - T)/T_\lambda]$. The straight line has a slope 0.80. The errors are discussed in the text.

that our data can be fitted by the equation $(\eta_\lambda - \eta)/\eta_\lambda = \alpha\varepsilon^x$, with $\eta_\lambda = 27.377\ \mu\text{P}$ and $x = 0.80 \pm 0.03$.

The filled points in Fig. 8 are data obtained with the first method and the open dots are results obtained with the second method. The main contribution to errors in the first method is due to the measurement of the resonance width, and can be $\pm 0.1\%$, as noted above. With the second method, used very near T_λ, the main contribution comes from the error in ρ_n. The normal density is in fact calculated using a measured temperature defined within $\pm 4\ \mu\text{K}$. From (14) we have for the maximum relative error $|d\eta/\eta| = |d\rho_n/\rho_n| + 2|dv_0/v_0|$. Using the formula of Tyson and Douglass,[12] $\rho_n = \rho[1 - 1.43(T_\lambda - T)^\alpha]$, where $\alpha = 0.666$, we get

$$|d\rho_n/\rho_n| \simeq |d\rho_n/\rho| = 0.73\varepsilon^{\alpha-1}|d(T_\lambda - T)|$$

In our case $|d(T_\lambda - T)| \leq |dT_\lambda| + |dT| \simeq 8 \times 10^{-6}$ and $|dv_0/v_0| = 3.5 \times 10^{-5}$. For our maximum error we have therefore $|d\eta/\eta| = 7 \times 10^{-5} + 5.9\varepsilon^{\alpha-1} \times 10^{-6}$. At $\varepsilon = 10^{-5}$ the term due to ρ_n is 2.7×10^{-4}, which is of the same order as $(\eta_\lambda - \eta)/\eta_\lambda$. With an improved thermometry we could get, at the limit, $|dT| = |dT_\lambda| \simeq 1\ \mu\text{K}$. With such an accuracy, $|d\eta/\eta| = 1.4 \times 10^{-4}$ at $\varepsilon = 10^{-5}$.

5. CONCLUSIONS

We have proved that the vibrating wire is a good instrument to investigate the behavior of viscosity at the λ transition. We think that Ahlers' suggestion is strongly supported by the result of the present experiment. The viscosity η reaches its value η_λ at T_λ following the relation $(\eta_\lambda - \eta)/\eta_\lambda = A(1 - T/T_\lambda)^x$, and because $x < 1$, the derivative $d\eta/dT$ should be infinite at T_λ. At present we must use an extrapolated value for η_λ and only the data below the superfluid transition. With improved cryogenics we hope to reach for $T > T_\lambda$ the same accuracy reached at present for $T < T_\lambda$. It should be possible in further experiments to get a definite conclusion on the small jump in η observed at T_λ, compare the x exponent below and above the transition, and investigate the whole λ line under pressure.

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