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Playing with the Atwood machine

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Introduction

Few students probably know that the Atwood's machine they meet in introductory physics textbooks dates back to 1784 and was designed for educational purposes.

Its inventor, Rev. George Atwood was in fact well known at his time for the demonstrations which illustrated his lectures.

His most famous machine was designed in order to perform experiments in kinematics and dynamics, with minimum friction, applying a constant force but avoiding too large velocities and too short time intervals.

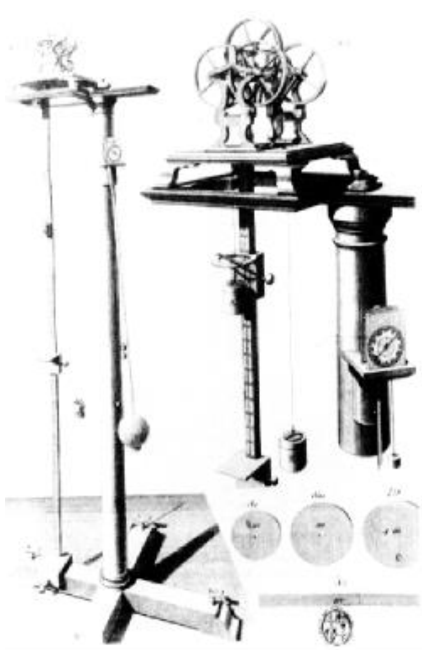


Figure 1

Fig. 1 shows the original device designed by Atwood. A rather complicated system of wheels was necessary in order to obtain a very low friction mounting for the pulley, whose moment of inertia was not negligible.

The five quantities involved in the demonstrations were the mass that was accelerated, the accelerating force, the distance travelled during acceleration , the time of acceleration and the velocity gained from rest during this time .

The distance was measured directly on the vertical scale whereas the time was measured by adjusting the distance so that it corresponded to an integral number of seconds measured by a clock beating seconds.

The final velocity was measured by allowing the extra mass to be picked off when the heavier body fell through the hole in the bracket on the vertical scale, thus obtaing a system in equilibrium which travelled at constant speed. An appropriate choice of the values of the distance of acceleration was obviously critical for the successful performance of the demonstration.

The aim of Atwood's demonstrations was essentially to show the form of the proportionality between any two of the variables with the others being held constant.

Nowaday the Atwood machine is often considered an historical relic (vestige) good only for virtual-experiments in physics textbooks. We will prove instead that this device, once it is properly interfaced with a PC, may offer many new interesting opportunities for teaching dynamics in the modern physics laboratory.

As any other mechanical system, the Atwood machine may be described by different models, depending on the features of the particular device we are considering.

A general description of the device might be: "two masses hanging from the ends of a non-stretching and light string passing over a pulley which rotates with negligible friction on its horizontal axis".

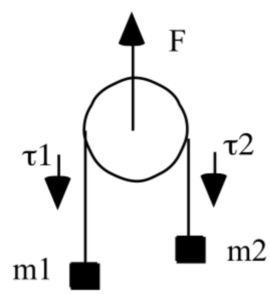


Figure 2

If we assume that the mass of the pulley and of the string are negligible, that the string is inextensible, and that there is no friction, we have the simplest description of the system.

By applying Newton's law to the two masses, separately, we get:

$$\tau - m_1 g = m_1 a$$

$$m_2 g - \tau = m_2 a$$

Therefore the acceleration of the system is :

$$\alpha = g\,(m_2 - m_1)\,/(m_1 + m_2)$$

For our purposes we shall assume it to be positive for clockwise motion of the pulley (i.e. $m_2 > m_1$).

The effect of the pulley mass

If we take into accout the mass of the pulley (or its moment of inertia I) our model needs to be revised.
We may equate the torque applied by the net driving force $T=(m_2-m_1)gR$ to the rate of change of the total angular momentum $\partial L/\partial t$ (R being the pulley's radius).
For the pulley the angular momentum is $L_p=I\omega=Iv/R$ (we assumed no slip between the string and the pulley, i.e. $v=\omega R$).
The angular momentum of the two masses are $L_1=m_1vR$ and $L_2=m_2vR$, and therefore the total angular momentum is $L=L_p+L_1+L_2=(I/R^2+m_1+m_2)vR$.
Defining an effective mass of the pulley as $M = I/R^2$, we get

$$T = (\mu_2 - m_1) g R = \partial L / \partial t = (M + m_1 + m_2)\partial v / \partial t R$$

that gives the acceleration

$$a = (\mu_2 - m_1)g /(m_1 + m_2 + M)$$

This is essentially Atwood's original model, where friction was neglected.

The effect of friction

If a (constant) friction force F_f is present, the acceleration reduces to:

$$a' = a - F_f/(m_1+m_2+M)$$

In order to evaluate friction we may measure the actual acceleration a' for different values of the driving force $F =(m_2-m_1)\,g$.
In this case we expecta linear behaviour of the function F(a'): $F= F_f + (m_1+m_2+M)\,a'$, so that plotting F versus a' we obtain F_f as the intercept at a'=0, while the slope gives us the value of the total inertial mass $M+m_1+m_2$:
An example of the results obtained in this experiment is shown in figure 3.
Here we used a thin aluminum pulley with radius R=8 cm, loaded by two equal weights of 80 g, and 2 sets of 4 small masses m* of 1 g each.
Four measurements of the acceleration are taken after displacing one of the small masses from one side to the other: in this way the driving force takes successively the four values $F = 2m^*g$, $4m^*g$, $6m^*g$, $8m^*g$, while the total inertial mass remains constant.

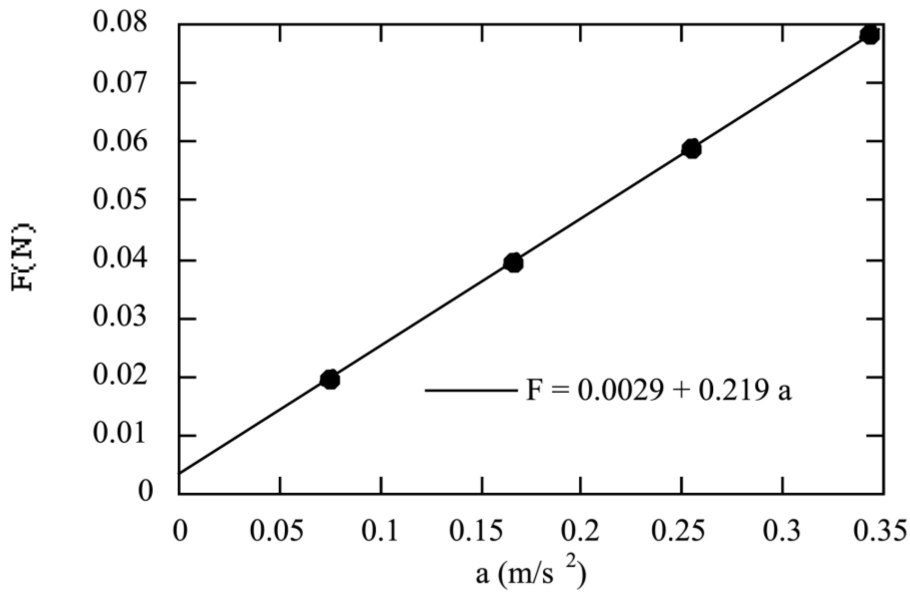


Figure 3

Here the difference between the total inertial mass (m_1+m_2+8m^*+ M=219 g, measured by the slope in the plot) and the total load (m_1+m_2+8m^*= 168 g) gives the effective inertial mass of the pulley M = 51±1 g.
From the intercept we can estimate $F_f \approx 3\ 10^{-3}$ N.

The force measured by the probe

The force F measured by the probe (subtracting the constant weight of the pulley) equals the sum of the tensions ($\tau_1$and τ_2) in the two sides of the string.

If the pulley is blocked, the force equals the total weight of the two masses:

$$F=(m_1+m_2)g$$

If the heavier mass (m_2) is blocked, the force F is twice the weight of the lighter one:

$$F = 2\tau = 2\ m_1\ g$$

If the lighter mass (m_1) is blocked the force F is twice the weight heavier one:

$$F = 2\tau = 2\ m_2\ g$$

When the system is left free to move the force is less than the weight of the two masses; some algebra leads to:

$$F= \tau_1+\tau_2 = g\ \{4m_1m_2 + M(m_1+ m_2)\} / (m_1+m_2+M)$$

The Atwood experiment: from constant acceleration to constant speed

We used two equal masses (80 g each) and we placed, over one of the two, an extra mass ⊗m=5.2 g (flyer) that is picked off by a fork during its fall. The pulley has an effective inertial mass M=51 g, so that the total inertial mass is 0.216 Kg, and the calculated acceleration is a=(0.0052/0.216=0.236) m/s^2.
Figure 4 reports two records of the dynamics. The difference between the calculated and the *mean* measured value a'=0.215 m/s^2 may be attributed to the friction:

$$a' = a - F_f/(m_1+m_2+M) = (0.236 - 0.003/0.216 = 0.213)\ \text{m/s}^2.$$

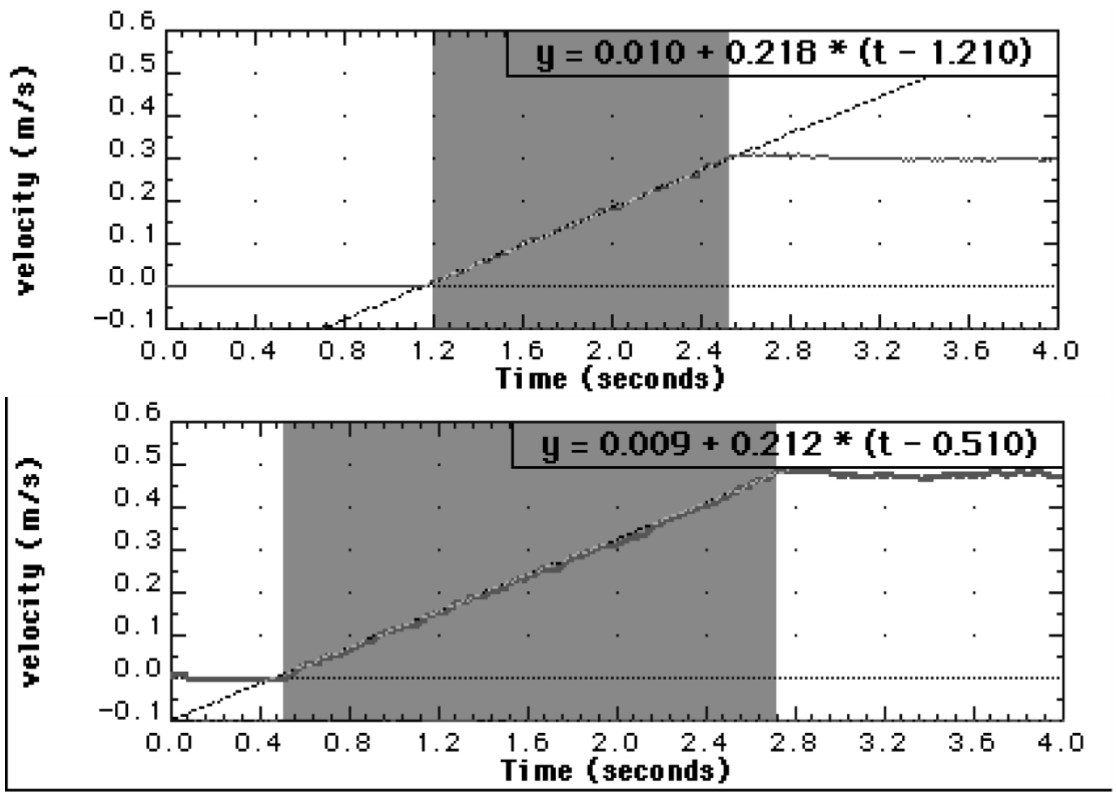


Figure 4

The force measured by the sensor

The force measured by the sensor changes according to newton's law: at first, when we block the lighter mass, we measure an average value F = 1.662 N, to be compared with the expected value $F = 2m_2g$ =1.666 N; then, when the system is accelerating, we measure F=1.613 N, to be compared with the expected value $F= [4m(m+\Delta m)+M(2m+\Delta m)] / (2m+\Delta m+M)g$ = 1.616 N;
finally, when the extra mass Δm is picked off, we measure F=1.563 N to be compared with the expected value $F=2m_1g$=1.568 N.

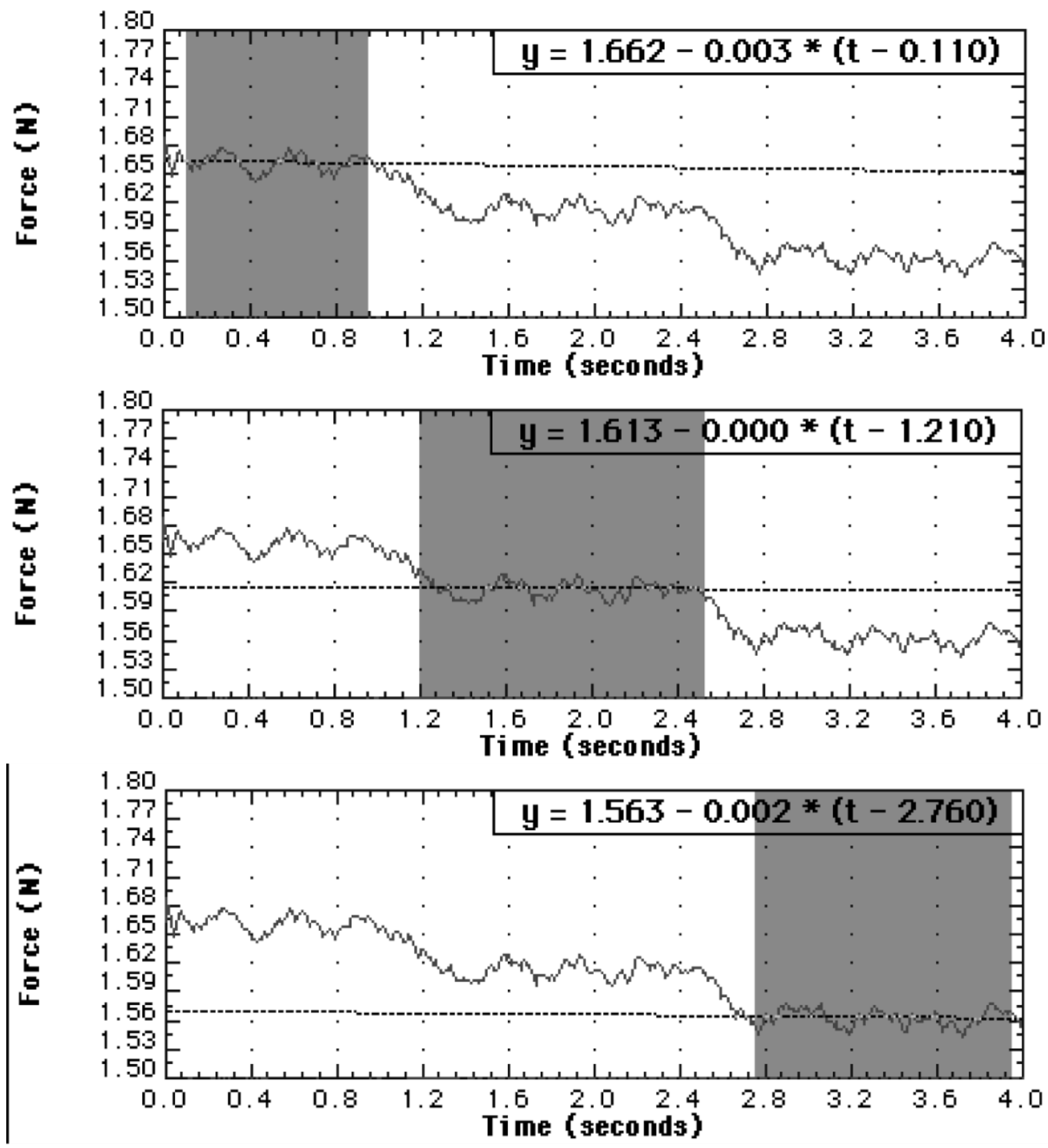


Figure 5

A variation on the Atwood experiment

If we repeat the Atwood experiment using *unequal* masses (being lighter the one to which we add the extra mass), we may obtain an oscillating motion:

For example using m_1=81.8 g, m_2=80 g, and Δm=5 g, we have a driving force $F_1=0.0032\cdot 9.8\approx 0.03$N before the extra-mass is picked off, and $F_2=-0.0018\cdot 9.8\approx -0.018$N after.

With an inertial mass of 51 g for the pulley, the total inertial mass is 213 g and 218 g respectively .

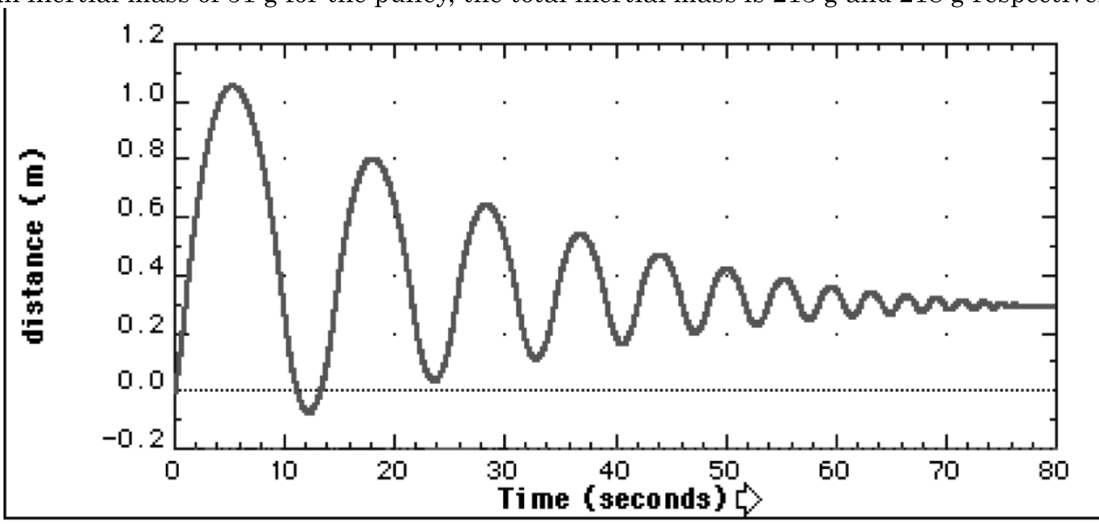


Figure 6

The record of the position versus time for this oscillation, shown in figure 6, might suggest at first sight a damped *harmonic* motion... but as soon as we plot the velocity versus time we realize that this is not true (figure 7): the velocity does changes *linearly* with time, with an acceleration that depends on the two different driving forces.

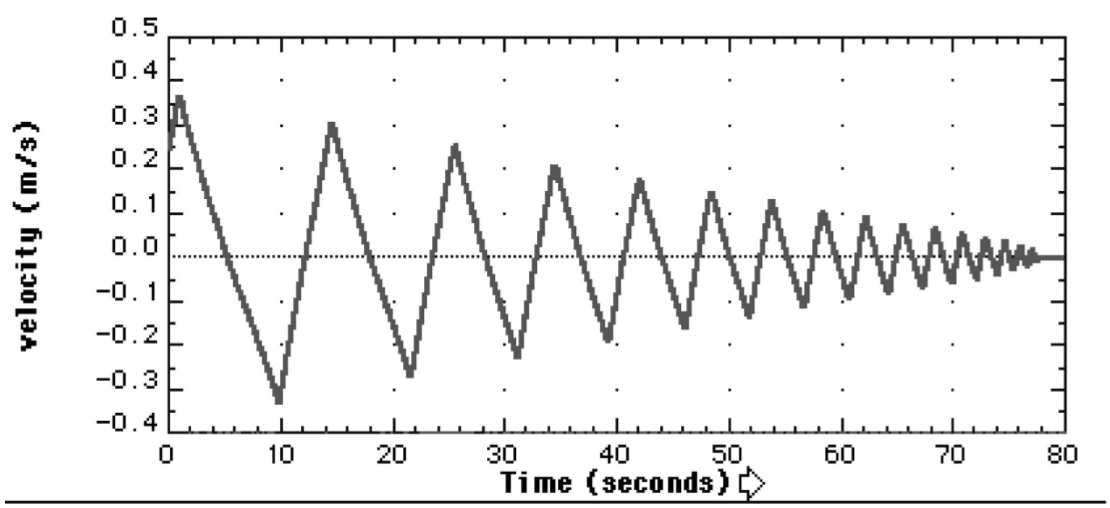


Figure 7

Figure 8 reports a linear fit of v(t) in a portion of the motion with the extra mass: the positive slopes (corresponding to larger measured force) give a value for the acceleration which is close to the expected one (a=0.144 m/s^2).

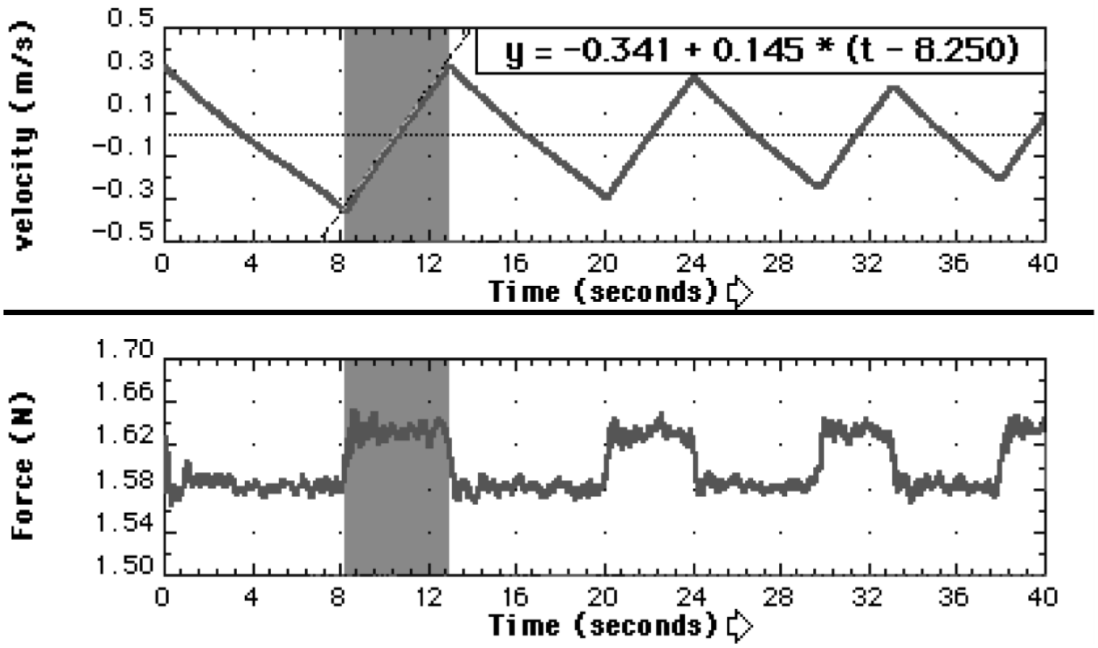


Figure 8

Effect of string friction

If we look at the velocity versus time plot more in details, we realize that the *negative* slope intervals (describing the motion with extra-mass picked off) exhibit slightly different slopes in the positive and negative velocity portions: this is due to the friction of the string.

The expected acceleration would be –0.083 m/s^2: therefore we observe a velocity with a *faster decreases* (the rate is 0.085 m/s^2 when driving force and friction are both directed upward) and a *slower increase* (the rate is 0.070 m/s^2 when the body accelerates upward and the friction force is directed downward), as shown in figure 9.

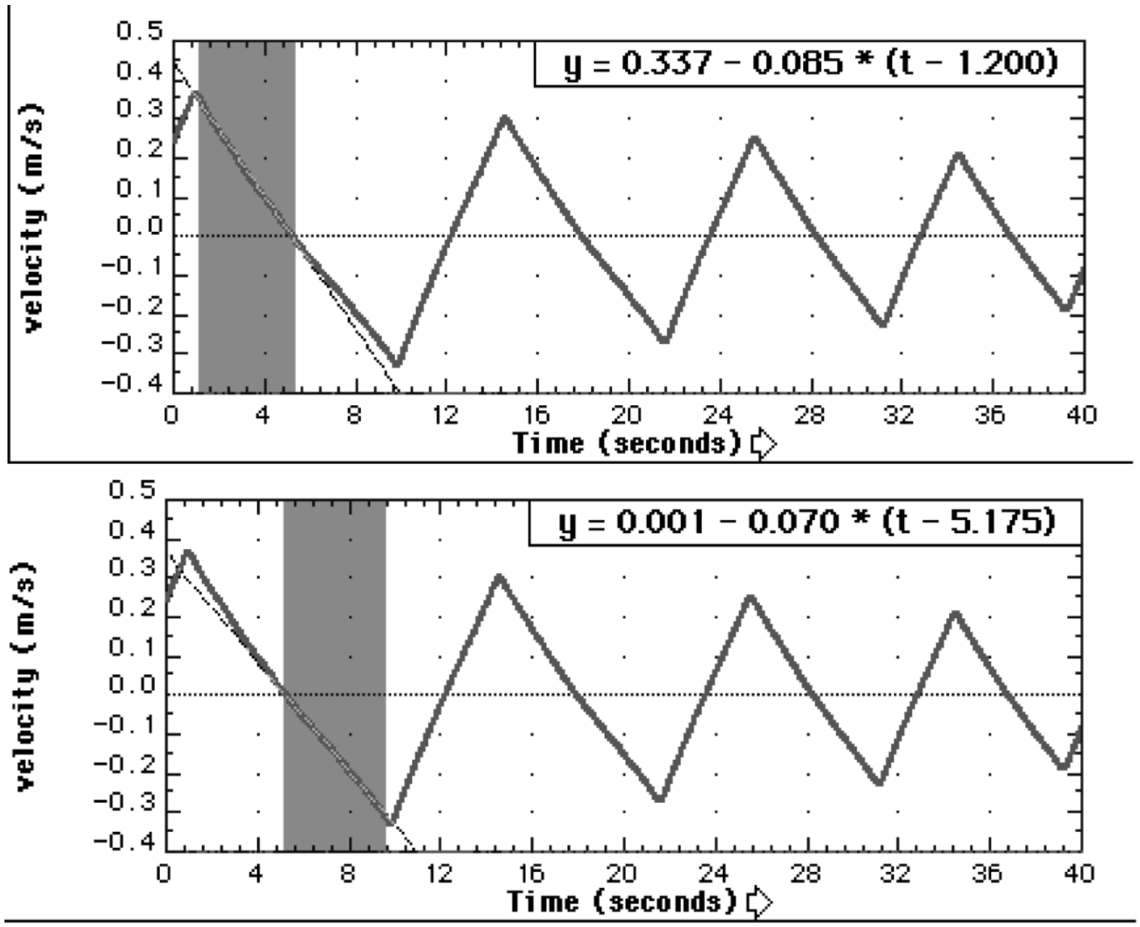


Figure 9

This difference may be explained assuming an extra friction force acting on the string of about 0.002 N.

Another variation of the Atwood machine: oscillations driven by buoyancy force

Instead of using a fork to suddenly change the net driving force, we may exploit the *buoyancy force* (or Archimede's force) that show up when we dip the heavier mass into a liquid (e.g. water). In other word: if the excess mass is smaller than the mass of the water displaced by the heavier body when it is completely submerged, the net force applied to this body by the string tension and by the gravitational field will be directed upward.

A record of such resulting oscillation is shown in figure 10, obtained with a loading of m_1=113 g and m_2=121 g, and a total inertial mass of 285 g.

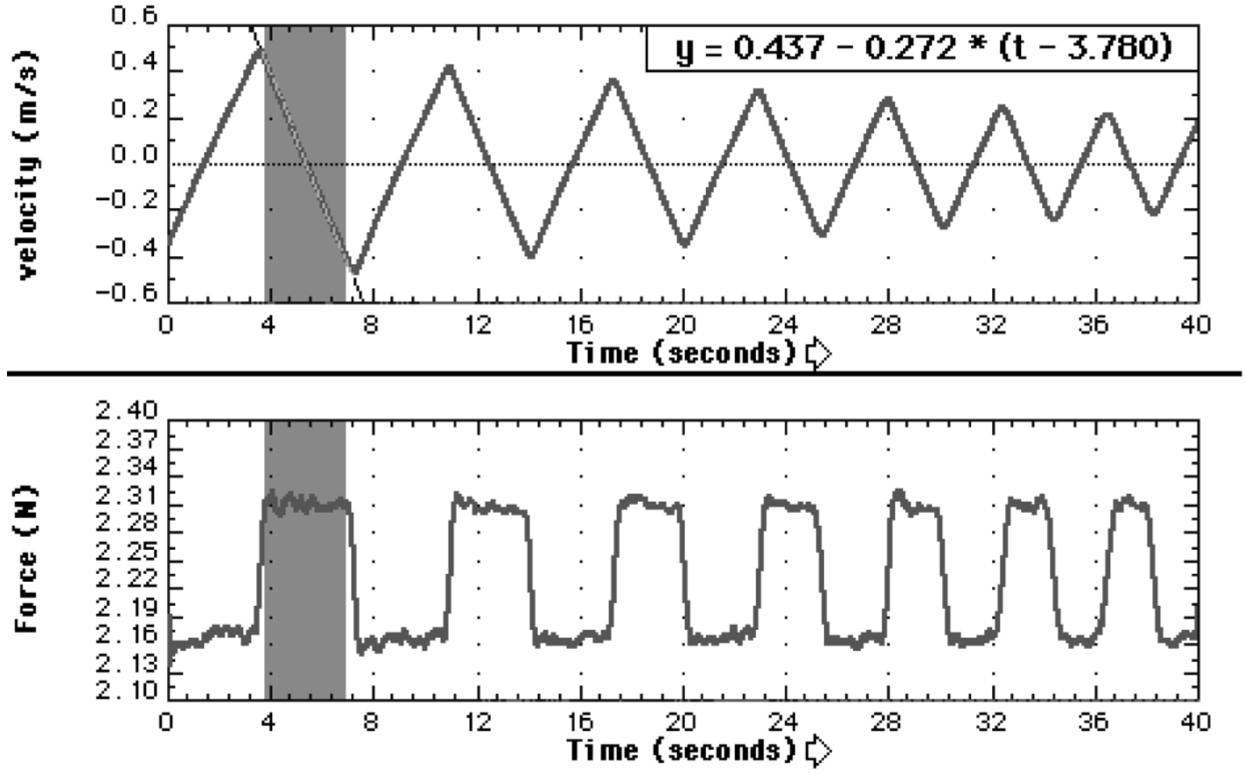


Figure 10

The calculated acceleration for the body out of the water is a=0.008·9.8/0.285=0.275 m/s^2 , and we measure a=–0.272 m/s^2, with a difference that may be explained by a friction force of 0.002 N.

When the body is underwater the acceleration changes sign and it takes the value a=0.248 m/s^2 (figure 11). This means that the net force acting on the submerged body is roughly equal to the opposite of the previous one. When the oscillation stops the heavier body rests in equilibrium approximatively half-submerged.

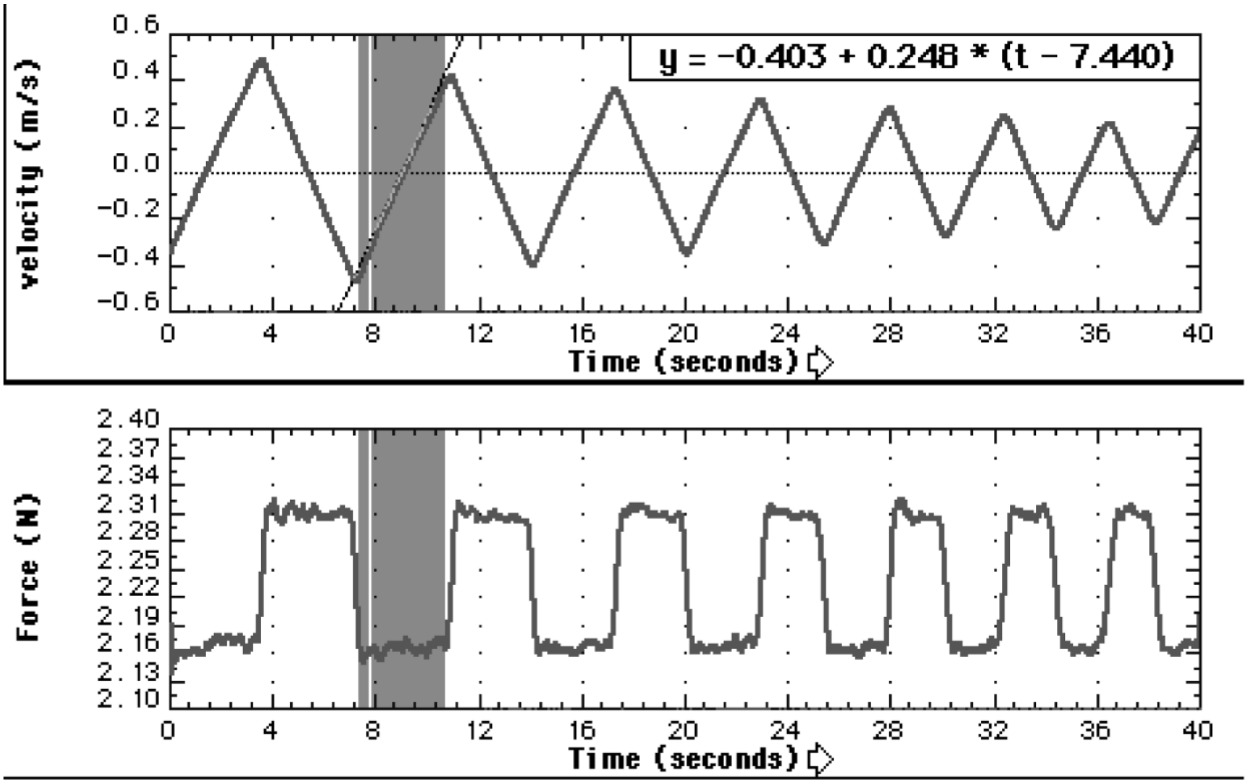


Figure 11

In fact the body is a cylinder 9 cm long with a diameter of 1.5 cm, whose volume is V≈16 cm^3, and when it is floating with the water surface at half of its height the wheight of the displaced water equals the excess mass (8 g).

Figure 12 shows a record of the whole oscillation.

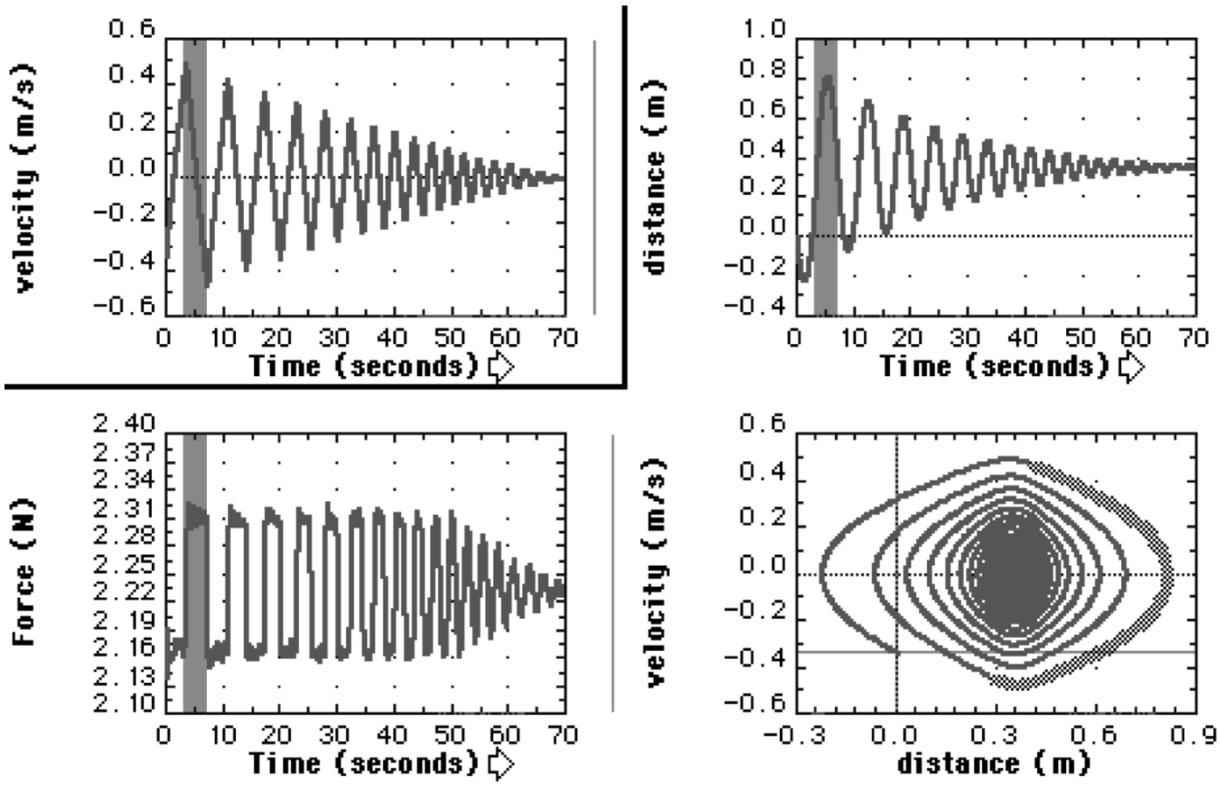


Figure 12

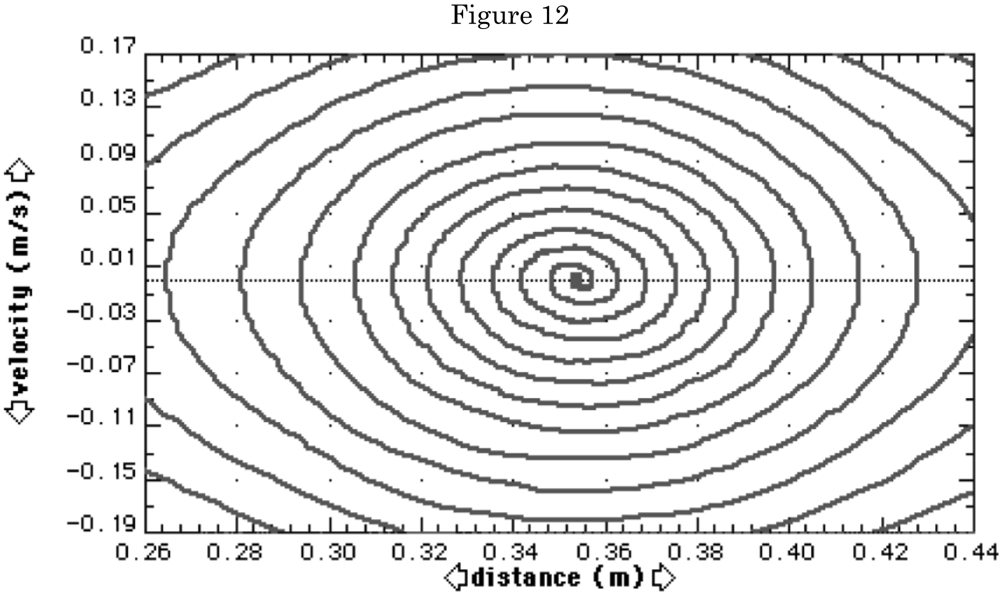


Figure 13

When the cylinder is always *partially* submerged the oscillation becomes a damped *harmonic* motion, as shown in figure 13. Here the restoring force is the difference between the Archimede's force $F_A = \pi r^2 x \rho g$ (where x is the submerged lenght and ρ the density of water) and the gravitational force due to the excess mass $F_e = \Delta mg$.

The frequency of the harmonic motion is determined by the total inertial mass and by the e*ffective elastic constant* $k = \pi r^2 \rho g$: the greater is the liquid density, the higher is the frequency

Let us suppose to start with a solid cylinder of radius r whose lower base touches the free surface of a liquid contained in a cylndrical vessel of inner radius R. Let x be the lower base displacement with respect to the initial liquid free surface level, and y the upward displacement of the liquid surface due to the cylinder downward motion: the following relation holds

$$y(R^2–r^2)=xr^2$$

If x' is the distance between lower base and the actual level of the liquid free surface (moving up during cylinder dipping) we have $x'=x+y$ and thereore:

$$x'=x/(1–r^2/R^2),$$

Which holds until the upper base reaches the liquid free surface.

This means that the effective *"elastic constant"* of the oscillatory motion of the floating cylinder is affected by the finite radius of the vessel, becoming $k=\pi r^2\rangle g/(1–r^2/R^2)$.

In our case (r=0.75 cm and R=5 cm) such correction is 2.3% on k, i.e. 1% on the frequency $\Omega=\sqrt{(k/m')}$

Here m' is m_1+m_2+M+M', where M' is the hydrodynamic mass (that being a function of x should give an anharmonic contribution)

One more vatiation on Atwood oscillation: the case of the cone.

If we use, instead of a body with *constant cross section*, a body with a different geometry we may expect some change in the dynamics

An interesting case is that of a body with conical shape. (the case of pyramidal shape is equivalent)

Suppose our heavier mass is shaped as a twin cone(two identical cones attached at their vertex) and that the lighter mass is chosen in order to keep the twin cone in equilibrium *half-submerged* in a water bath.

The buoyancy force in this case is $F_A=\pi r^2 h\rho g/3=\pi(tg\alpha)^2 h^3\rho g/3$, where r, h and α are the cone radius, height and angle of aperture, respectively (figure 13).

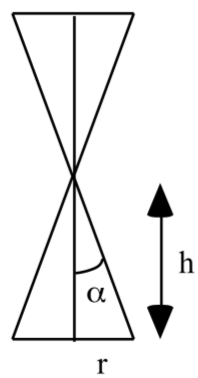


Figure 13

Being the system in equilibrium, the sum of the buoyancy force and the string tension is equal to the twin cone weight. If we displace upward vertically the twin cone of an amount +x, the buoyancy force changes of a quantity $\Delta F_A=-[\pi(tg\alpha)^2\rho g/3]x^3=-kx^3$.

Therefore our modified Atwood machine will behave as an *anharmonic oscillator*, whose frequency should be proportional to the oscillation amplitude.

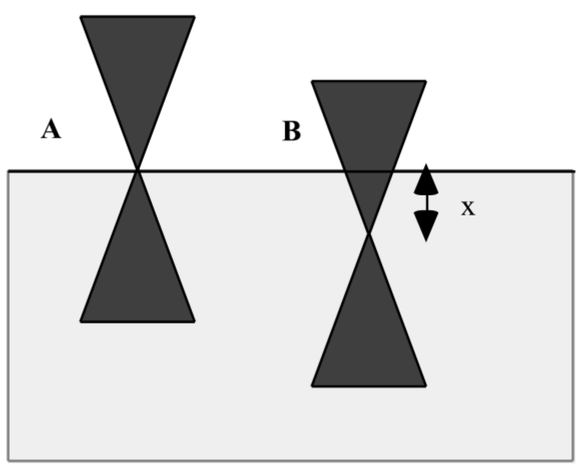


Figure 14

A record of the motion of the twin cone is shown in figure 14, where it is apparent that the pseudoperiod increases as the oscillation amplitude decreases due to viscous damping.
Also we see that the plot of force versus distance follows a cubic line.

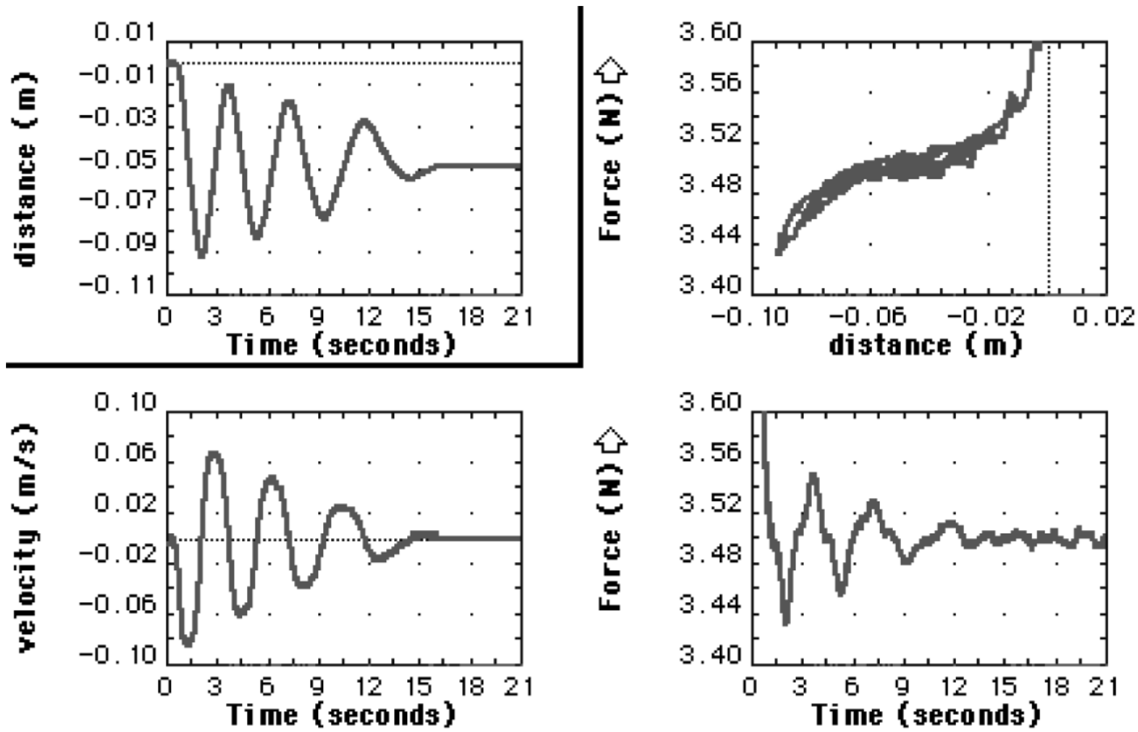


Figure 14

In order to test the expected linear dependence of frequency on amplitude, we measured the time intervals for each half pseudoperiod and the corresponding excursions.
Figure 15 shows the resulting plot.

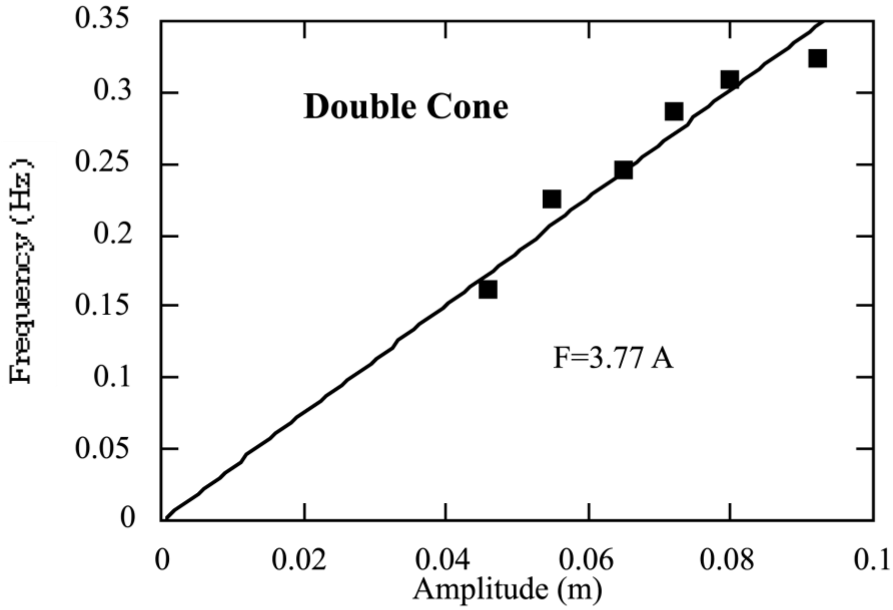
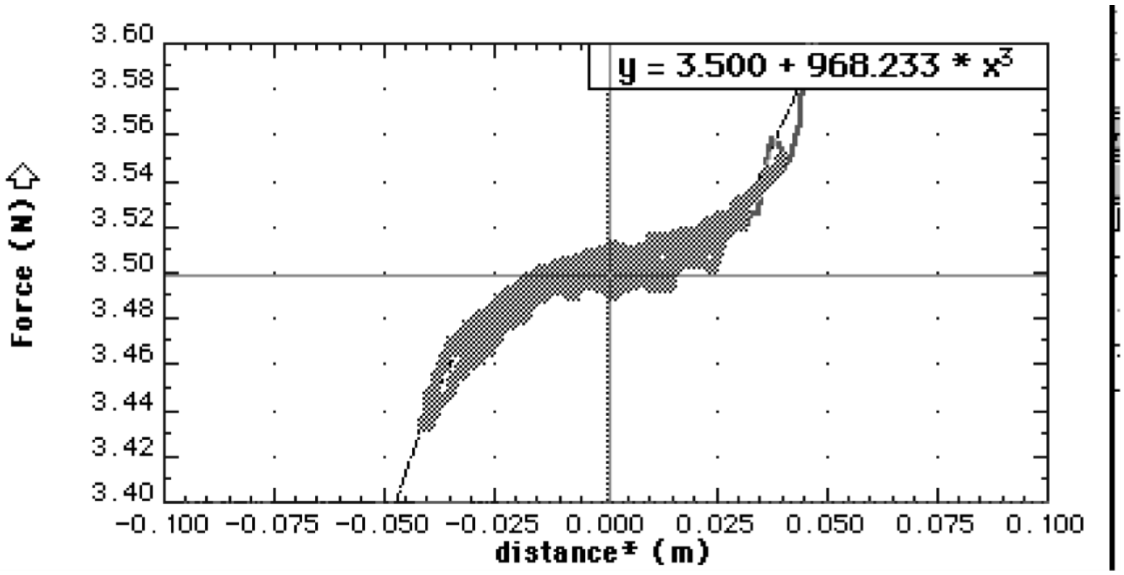
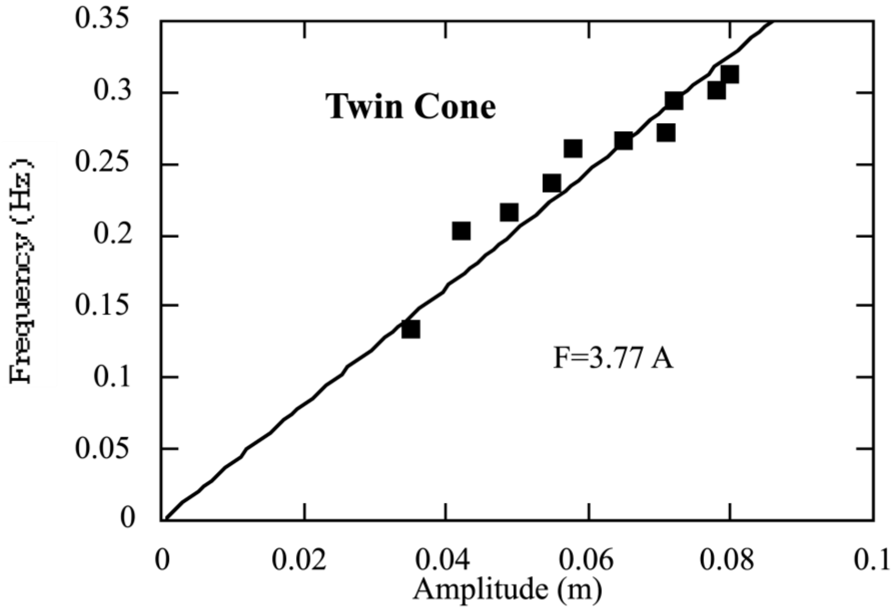
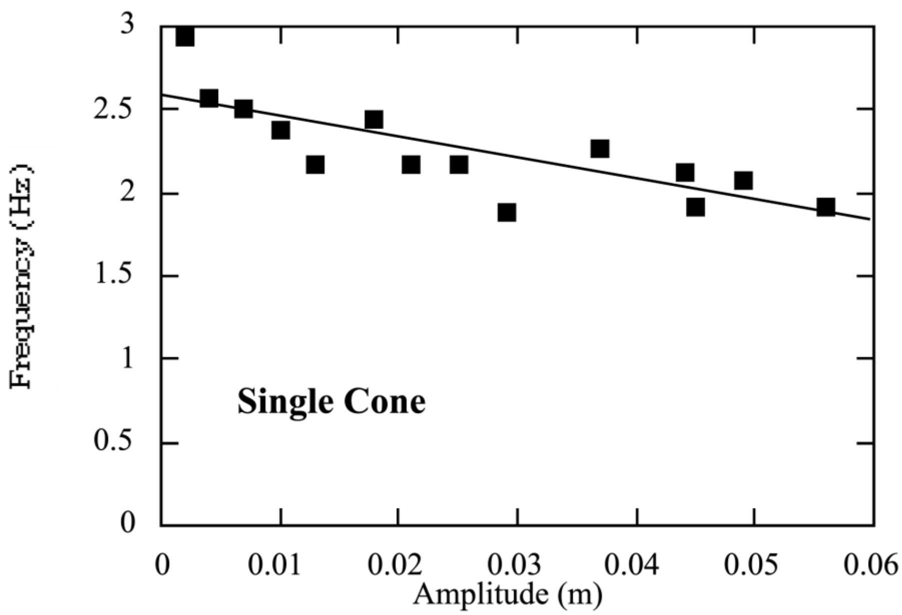

Double Cone
F=3.77 A
Frequency (Hz)
Amplitude (m)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1

Figure 15



y = 3.500 + 968.233 * x³
Force (N)
3.60
3.58
3.56
3.54
3.52
3.50
3.48
3.46
3.44
3.42
3.40
-0.100 -0.075 -0.050 -0.025 0.000 0.025 0.050 0.075 0.100
distance* (m)


Twin Cone
F=3.77 A
Frequency (Hz)
Amplitude (m)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.02
0.04
0.06
0.08
0.1


Single Cone
Frequency (Hz)
Amplitude (m)
3
2.5
2
1.5
1
0.5
0
0
0.01
0.02
0.03
0.04
0.05
0.06