Physics of the Seesaw

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The idea behind this article was inspired by a previously published *TPT* note on harmonic and anharmonic oscillators,[1] in which the oscillations of a bar on a cube were briefly discussed. We thought these oscillations deserved quantitative experimental investigation and that automated data acquisition could allow the data to be gathered.

We decided to start the investigation by studying the motion of a real seesaw and exploiting the modern technologies available nowadays. The data-acquisition system we used is a graphing calculator (TI-89 or TI-92) connected to CBL and to a motion detector. We chose this system because of its portability, which allows measurements to be easily performed in real-life contexts. Many systems available on the market, however, can be used for the purposes of this investigation.

Studying a Real Seesaw

A simple seesaw can be made in a school gymnasium using a ladder on a flat balance beam (see Fig. 1). Students are first invited to observe its motion unloaded in order to detect special features or regularities. They can then observe what happens if two of them are sitting at the ends of the ladder and compare the motion of the system in the two different situations. They will easily find out that in both cases, the time between successive oscillations decreases with time, and that the time in the case of the *loaded* seesaw is longer than that of the *unloaded* one.

Fig. 1. Students on the seesaw.

Using the data-acquisition system connected to a motion sensor placed under one end of the ladder, it is easy to collect position-versus-time data. The plot can be projected on the wall and compared with direct observations. The graph appears very much like that for harmonic motion, but this impression soon fades when the velocity and the acceleration are plotted versus time. It is easy to recognize that the motion is due to an approximately constant torque (leading to a constant angular acceleration) that changes its sign each time the ladder passes through its equilibrium position.

Students can also be invited to investigate what happens when some parameters are changed (for example, the total weight at each end) by looking at how the plots change under different conditions.

Studying the Seesaw in the Lab

Moving to the laboratory we can study the system under more controlled conditions. We investigate its motion by varying the basic parameters that can influence the time of one oscillation and checking the theoretical model with experimental data. One effect that had not been explored in the gymnasium is the influence of the shape of the pivot. This can be done in the laboratory by substituting the block with a cylinder. Therefore we will study two cases: the square pivot and the round pivot.

We developed a theoretical model that can account for the observed features of the motion that will allow us to make quantitative predictions to be checked on a small-scale seesaw in the school laboratory. To keep this note short, we shall only sketch the path leading to the model.

The unloaded seesaw on a square pivot

We here assume that the oscillation amplitude ϕ is small and that the pivot width d is much smaller than the bar length L (see Fig. 2). The equation of motion is $I\, d\omega/dt = \tau$, where τ is the torque, I the moment of inertia, and ω the angular velocity. The torque is provided by a component of the gravitational force F applied to the bar center $F = mg \cos\phi \approx mg$ (for small ϕ values), so that the restoring torque[2] is $\tau \approx -(d/2)\, mg\, \mathrm{sign}(\phi)$. The moment of inertia may be written $I = (1/12)mL^2 + m(d/2)^2 = (m/3)(L/2)^2 (1{+}3d^2/L^2) \approx mL^2/12$, for $d{<<}L$. Then the angular acceleration is $\alpha = \tau/I \approx (mgd/2)/\,(mL^2/12) = 6gd/L^2$ and the magnitude of the linear acceleration of the bar end is $a \approx (d\omega/dt)L/2 = 3gd/L$. The vertical component of the linear acceleration at small angles is $a_z = a\cos\phi \approx 3gd/L$, which is constant. We may therefore predict for the oscillation period:

$$T \approx 4\sqrt{2A/a_z} = 4\sqrt{2L/3gd}\sqrt{A}, \quad (1)$$

where A is the vertical amplitude.

In the lab we used a small-scale seesaw made of an aluminum bar of L = 80 cm and a metal block that provides two pivot edges separated by d = 4 cm, which satisfies our assumption of $d{<<}L$ within 5%. Figure 3 shows the data of the position, velocity, and acceleration versus time, as they appear on the screen of the graphing calculator.

By plotting the time of each oscillation versus $\sqrt{A}$ (see Fig. 4), we obtain $T = (4.7 \pm 0.3)\sqrt{A}$,

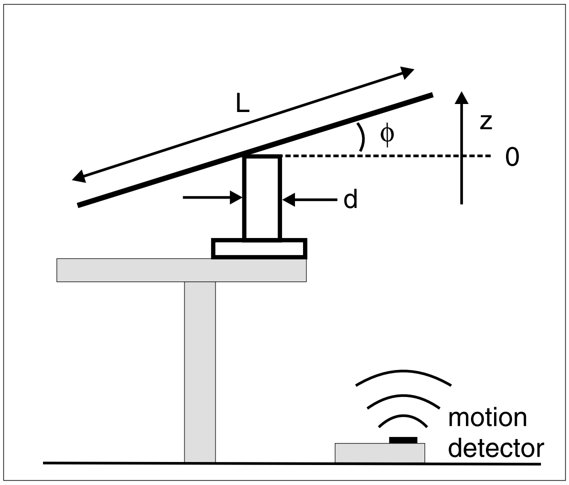

Fig. 2. Setup for seesaw with square pivot.

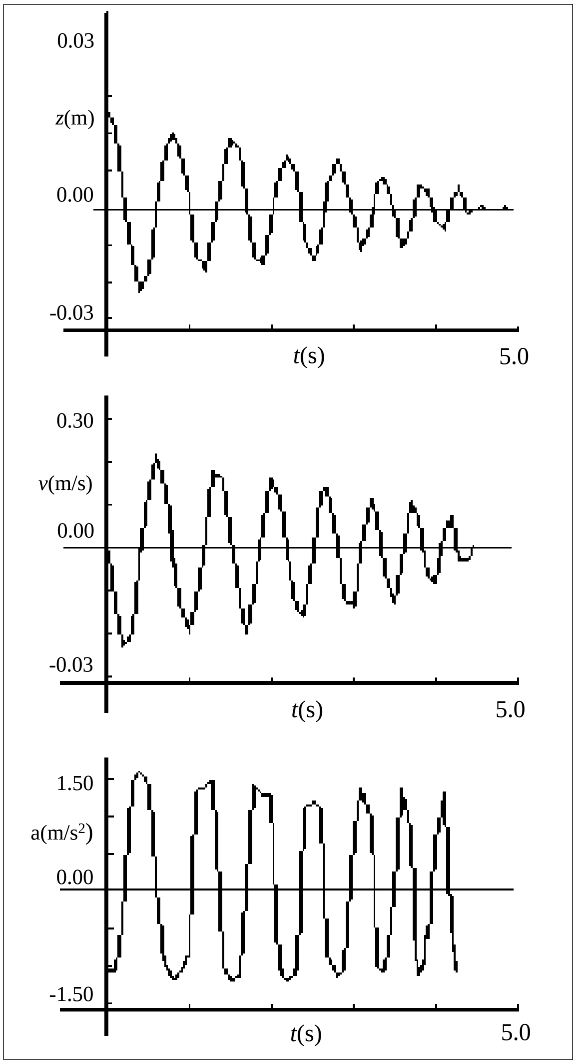

Fig. 3. Plots of position (*x*1), velocity (*v*2), and acceleration (*a*3) versus time *t* for square pivot.

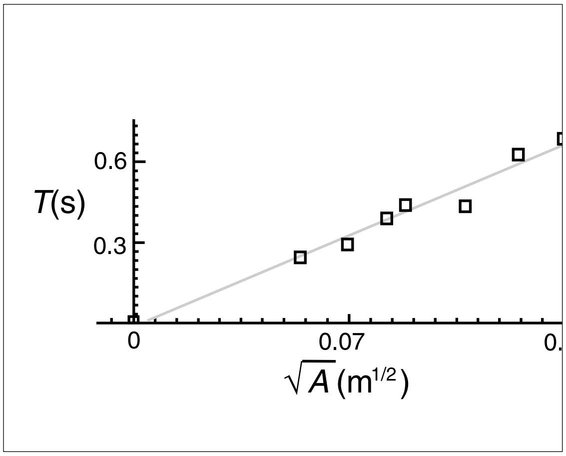

Fig. 4. Plot of period versus square root of amplitude (unloaded seesaw with *d* = 4 cm).

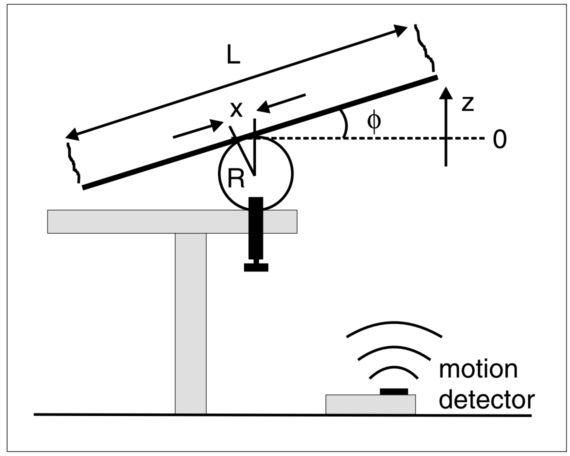

Fig. 5. Setup for seesaw with round pivot.

which agrees well with the dependence $T = 4.66\sqrt{A}$ calculated from Eq. (1).

By repeating the same procedure with a thinner pivot (d = 2 cm), we obtain a plot where the best fit line is $T = (6.7 \pm 0.3)\sqrt{A}$, in agreement with the prediction $T = 6.6\sqrt{A}$ provided by Eq. (1).

It is interesting to notice that Eq. (1) does not contain the mass m: our model therefore predicts that the period should not depend on the bar mass but only on its length. Such a prediction may be easily confirmed by simply placing a second identical bar over the previous one and recording the oscillations: we obtain exactly the same behavior.

What happens when we sit on the seesaw?

From observations of students on the seesaw, we have noted that mass-loading the seesaw does affect its motion; that is, heavier students produce a slower motion. When a mass $M >> m/2$ is added to each *bar end*, the torque becomes $\tau = F(d/2) = (d/2)(m + 2M)g\cos\varphi \approx Mdg$, and the moment of inertia changes into $I = m/3\,(L/2)^2(1 + 3d^2/L^2) + M(L/2 + d/2)^2 + M(L/2 - d/2)^2 = \{(m/3)(1 + 3d^2/L^2) + 2M(1 + d^2/L^2)\}(L/2)^2 \approx ML^2/2$. Therefore, the vertical component of the acceleration of the bar-end is $a_z \approx (Mgd)/(ML^2/2)(L/2) = gd/L$ and the period may now be written as:

$$T \approx 4\sqrt{2A/a_z} = 4\sqrt{2L/gd}\sqrt{A}. \qquad (2)$$

We can check the dependence of the period from $\sqrt{A}$ in the case of a bar of mass m = 250 g loaded with two masses M = 730 g at the ends. The magnitude of the slope of the best fitting line is 12 ± 1, in fair agreement with the value 11.4 calculated from Eq. (2).

The case of the round pivot

Using the same bar on a round pivot (e.g., a metal tube with a radius of a few centimeters clamped to a table — see Fig. 5), we may study the effect of changing the pivot shape. A simple model tells us that it will produce substantial effects on the features of the motion.

If R is the radius of the cylindrical pivot, the displacement of the contact point is $x = R\phi$, and therefore the torque is $\tau = xmg\cos\phi \approx -R\phi mg$, and the moment of inertia is $I(x) = (m/12)L^2 + mx^2 \approx mL^2/12$. Here we have assumed that ϕ is small and $x << L$. The angular acceleration becomes

$$\alpha = \tau/I_0 = -(R\phi mg)/(mL^2/12) = -(12gR/L^2)\varphi.$$

This equation is the same as that of a pendulum with effective length $\lambda = L^2/(12R)$ [at small amplitudes $\alpha = -(g/\lambda)\varphi$]; therefore, the oscillation is *harmonic* with period

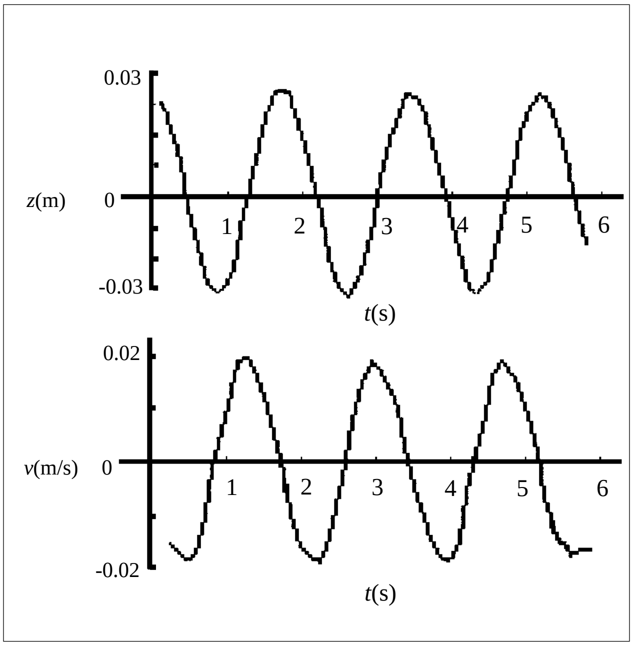

Fig. 6. Plots of position (*x*1) and velocity (*v*2) for round pivot.

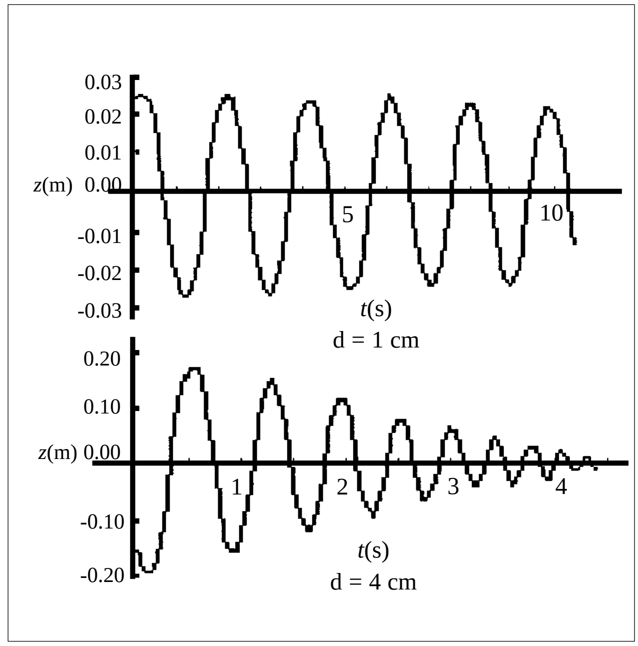

Fig. 7. Comparison between damping with two *d* values (1 and 4 cm) for the square pivot.

$$T = 2\pi\sqrt{(L^2/12gR)} = \pi L/\sqrt{(3gR)} \qquad (3)$$

Therefore changing from square to round pivot turns an anharmonic oscillation into a harmonic one.

Figure 6 shows the data of the position and velocity versus time obtained with an aluminum bar of L = 80 cm and a pivot of R = 10 cm. The experimental data provides a value for the period T = (1.47 ± 0.01)s, which agrees well with the value T = 1.46 s predicted by Eq. (3). An analysis of the motion of the bar on a round pivot loaded by suitable masses (M>>2m) leads to a predicted period which is $\sqrt{3}$ times larger than for the unloaded bar:

$$T = \pi L/\sqrt{(gR)}. \qquad (4)$$

A Side Issue on Damping

It can be observed that the damping increases as the diameter of the square pivot is increased, keeping all other conditions the same (see Fig. 7). As a limit case we can think of keeping the bar with one end on the floor and letting it fall down ($d = L$). Notice that in general damping is smaller for the round pivot seesaw than for the square pivot one.

We can try to explain this by looking to the motion of the *center of mass* in more detail. Plotting the vertical coordinate z_G of the center of mass versus time, we get a series of parabolas, similar to those describing the motion of a bouncing ball. But unlike the ball, most of the kinetic energy of the system is rotational and only a small fraction is translational. This fraction is proportional to the square of the velocity v_G of the center of mass when the bar passes through its equilibrium position $v_G^2 = 6z_G g\,(d/L)^2$. We can assume that only the translational kinetic energy is dissipated whereas the rotational kinetic energy is conserved, and is responsible for making the other end of the bar raise up. The energy loss takes place in fact via vibrations excited by impact of the bar against the pivot. Therefore, given that for a specific value of ϕ z_G is also proportional to d, the dissipated energy will be proportional to d^3. This can account for the noticeable dependence of damping upon the distance between fulcrums.

Here we have assumed that damping due to air friction can be neglected. This is not the case with the round pivot seesaw: in the absence of collisions of the bar with the pivot (assuming a perfectly round pivot, plane bar, and both rigid), damping can only be caused by air resistance and rolling friction. The fact that the damping increases with amplitude (and therefore with the velocity of the bar) indicates that the effect of air resistance is not negligible.

Comments

Looking at science textbooks, one gets the impression that oscillatory motions are mostly harmonic. This is not the case in reality. Real systems may show anharmonic features because of the presence of a constant gravitational field that is responsible for a restoring force of position.[1] In some cases a position dependence arises from the geometry of the system (as in the pendulum and in the seesaw on a round pivot), but in these cases the dependence can be approximated as linear only for small oscillations.

Recording of data over an extended period is especially important in investigating oscillatory motions. The automated data-acquisition system allows the recording of a large amount of experimental data that can be quantitatively analyzed later, even outside the laboratory, when the phenomenon is no longer available for observation.

Investigations that start from the study of phenomena in a nonsterilized context illustrate to the students that the conceptual tools developed in physics are special "ways of looking" at the phenomena around us, and not merely a series of abstract relations, so distant from everyday experience, as they often appear in physics textbooks.

Reference

1. R. Akridge, "Period and amplitude," *Phys. Teach.* **36,** 507-508 (1998).
2. In the following we simplify the notation by discarding the sign of torque and angular acceleration. (We only consider their absolute value.)