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Motion of bodies on an incline studied with PASCO MBL

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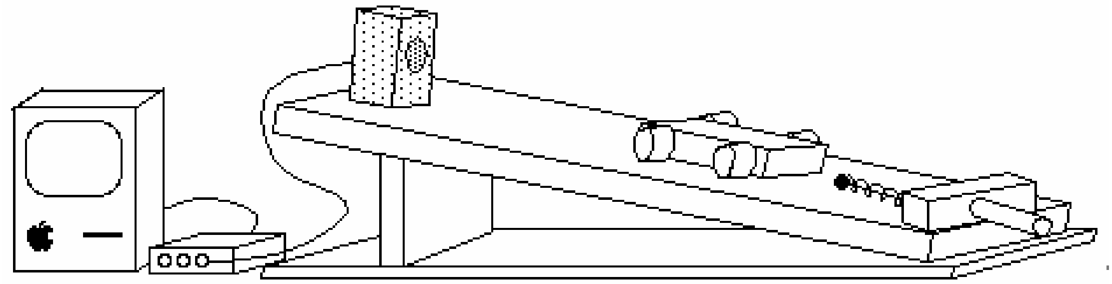
Motion of bodies on an incline

The investigation here described can be performed by using a motion sensor linked to an on-line acquisition system (e.g. a PASCO 500).



"Point-mass" cart

A special cart (like the PASCO ones) may have very little rotational inertia: in this case its motion on an incline is well described by a **point mass model**.

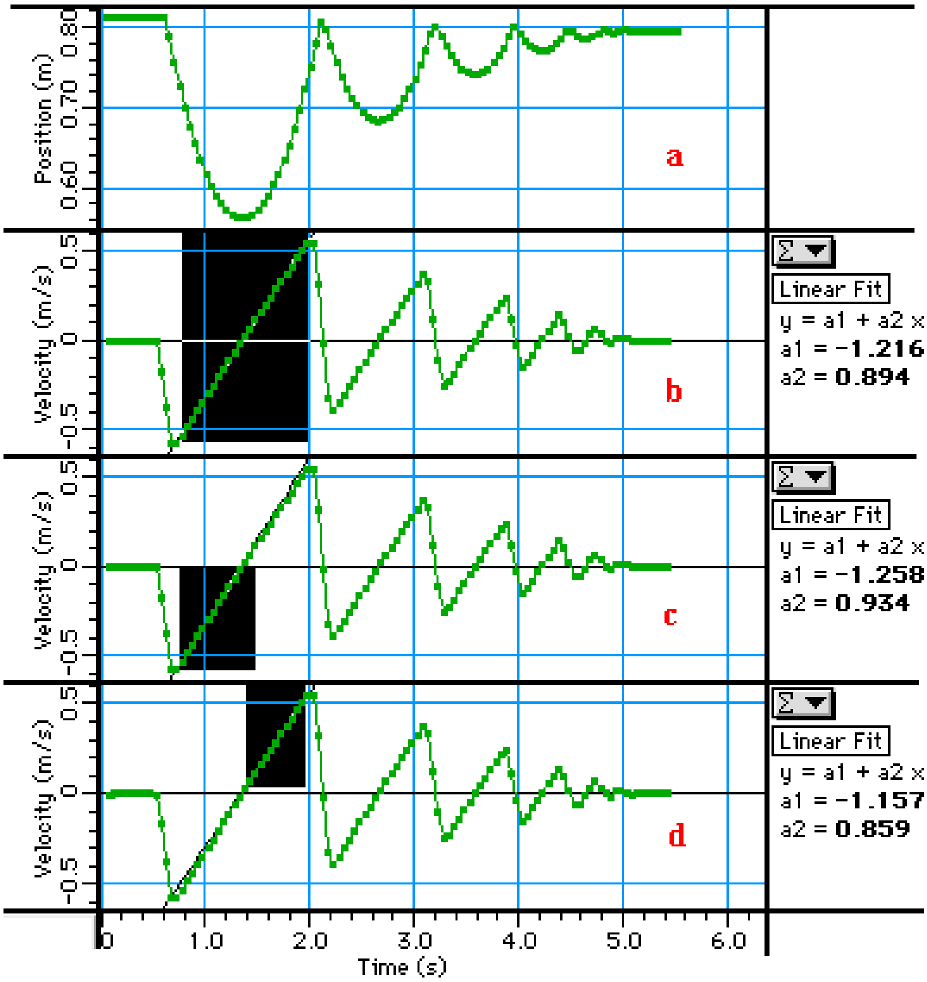


Figure 2. Plots of position and velocity vs. time for a "point-mass" cart rolling on incline.

If the incline slope is measured accurately enough (within 0,1 grades) the predicted value for the cart acceleration turns out to be in good agreement with the experimental data (within less than 1%).
Figure 2 shows the results obtained with a PASCO cart (m = 474g) pushed up an incline with 5.2 grades of slope and bouncing back at the lower end of the incline.
The motion sensor is fixed at the upper end of the incline. Therefore, the measured distance decreases as the cart rolls up.
The predicted value of the acceleration is $a = g.\sin\alpha = (0.89 \pm 0.02) m/s^2$ ..The experimental value, obtained by fitting the data from the velocity vs. time graph as in figure 2b, is $a = (0.895 \pm 0.005)$ m/s^2, which is consistent with the predicted one, but a closer analysis of the graph shows that the experimental value of the acceleration is not exactly the same for the upward and downward motions.

By separately selecting the two portions of the graph,one for the upward motion (figure 2c) andsecond for downward motion (figure 2d) respectively, one can calculate the two values $a_u = (0.934 \pm 0.005)$ m/s^2 and $a_d = (0.859 \pm 0.005)$ m/s^2, whose difference can be attributed to the presence of friction which adds to the gravity force in one case and subtracts in the other. By averaging the two values the acceleration component due to gravity can be calculated:
$a_m = (a_u + a_d)/2 = (a_g + a_{att} + a_g - a_{att})/2 = (0.90 \pm 0.01)$ m/s^2

Mixed sliding-rolling

By using a different cart whose rotational inertia is larger, we can show that the point mass model breaks down. If the cart wheels are homogeneous and geometrically simple, like cylinders, the inertia of each wheel is easily calculated ($I=(1/2)mR^2$) and the predicted value of the acceleration becomes

$$a = g\sin\alpha \ / \ (1+Nm/2M) = g\sin\alpha \ / \ \xi$$

where N is the number of the wheels each of mass m. Figure 3 shows the plot of the acceleration vs. sinα obtained by repeating the experiment with varying slopes, with an aluminium cart (M = 678g) with three cylindrical wheels (m = 46 g).
The comparison between the experimental data and the plots of the two functions gsinα and gsinα/ξ corresponding to the two theoretical models, shows that data are better represented by the one that takes into account the rotational motion of the wheels.

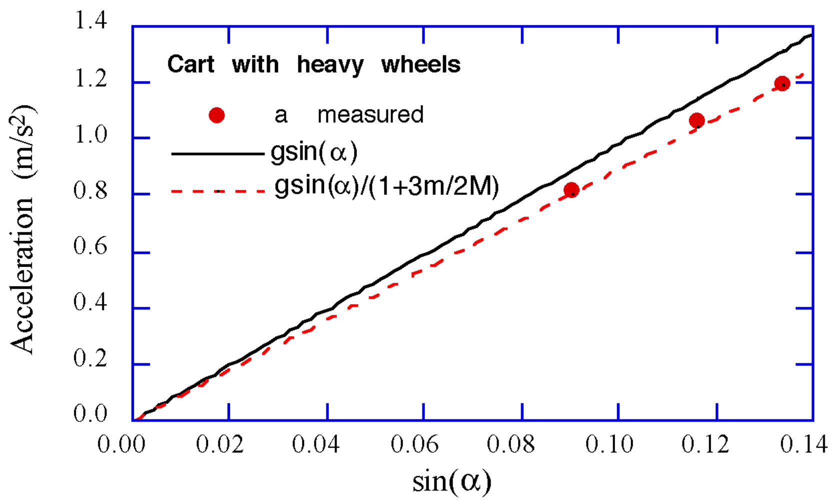


Figure 3: Plot of acceleration vs. sinα for a cart "with heavy wheels" rolling on incline .

Rolling bodies: a sphere on a track

Once the new model has been introduced it can be exploited for investigating the motion of variously shaped bodies (for instance cylinders or spheres, either solid or hollow) rolling on the incline. The factor ξ takes different values according to the mass distribution and this can be checked with experimental data. Figure 4 shows the data obtained by rolling up and down the incline a billiard ball of diameter D=57 mm, with α=5.2 degrees.

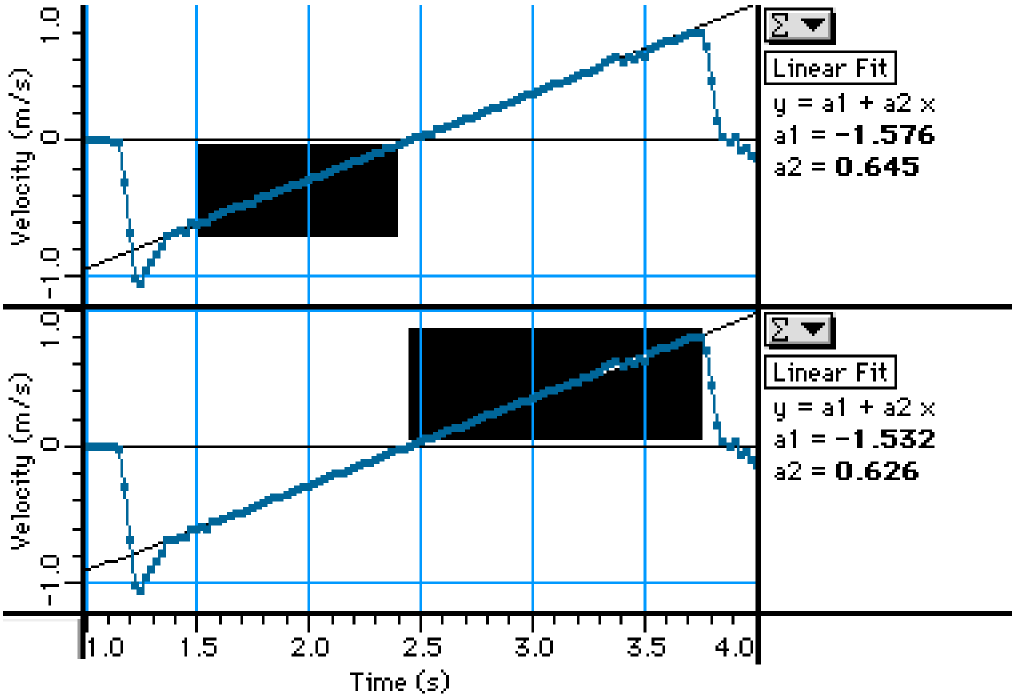


Figure 4: Plot of velocity vs. time for a billiard ball rolling on incline .

In this case (being I = $2mR^2/5$) the predicted value is $a = 5g\sin\alpha/7$= (0.635± 0.02) m/s^2 which agrees very well with the experimental value a = (0.636± 0.01) m/s^2 .

Effect of the track

A more complicated case is the investigation of the motion of solid or hollow spheres rolling on a track lying on the incline. Since the rotation radius does not coincide with the sphere radius, the ratio between the kinetic and the rotational energy depends upon the ratio between the sphere diameter and the track width.

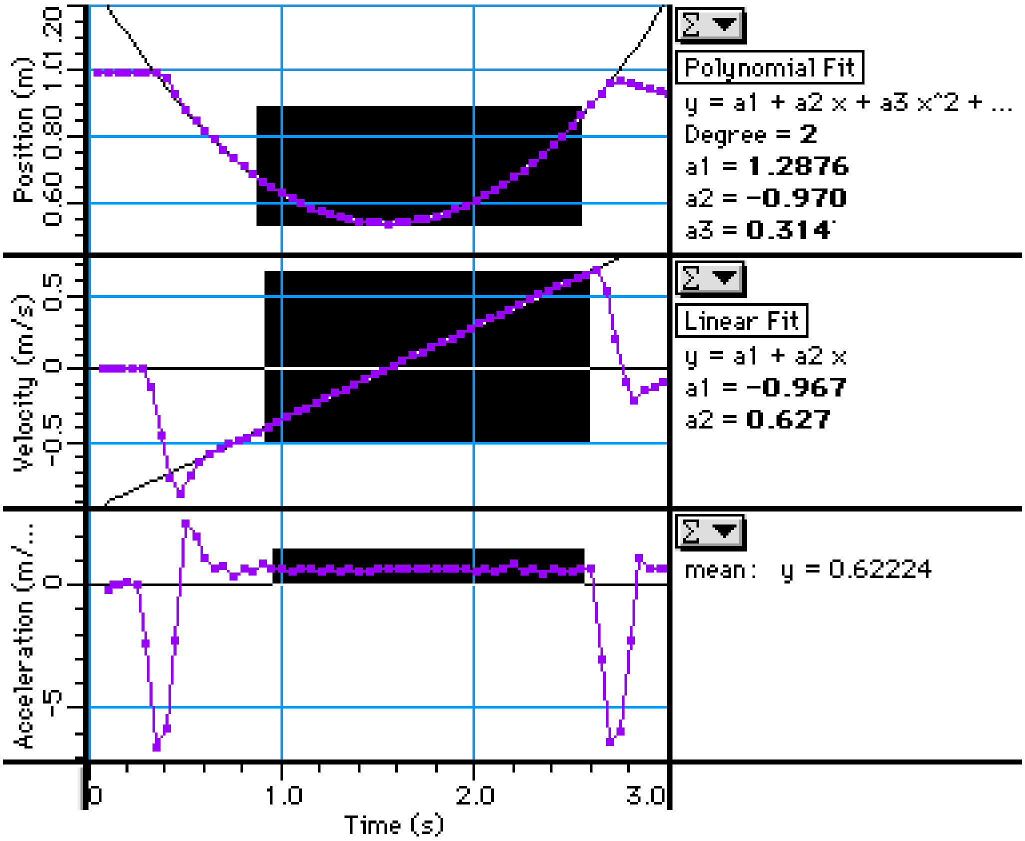


Figure 5: Position, velocity and acceleration vs. time for a billiard ball rolling on a track .

If a solid sphere of diameter Φ rolls on a track of width D we have

$$a = g\sin\alpha/\xi \quad \xi=1+2/[5(1-D^2/\Phi^2)]$$

According to the model the predicted value for the acceleration of the same billiard ball used in step three on a track of width Φ =16mm is $a=0.623 m/s^2$, which is in agreement with the experimental data shown in figure 5 (a=0.625 m/s^2).